



MolSpin

User Manual

February 2024

Luca Gerhards, Gediminas Jurgis Pažėra and Claus Nielsen
Carl von Ossietzky - University of Oldenburg



Contents

1	Installation	4
1.1	Linux	4
1.2	Windows	4
1.3	Testing your MolSpin installation	4
2	Getting Started	5
2.1	Input file basics	5
2.1.1	Overall structure: Objects and Object Groups	5
2.1.2	Comments	6
2.1.3	Case sensitivity and strings	6
2.1.4	Input file example	6
2.2	Running MolSpin	10
2.2.1	Commandline parameters	10
3	Specifying the spin system	12
3.1	Spins	12
3.2	Interactions	13
3.3	Transitions	13
3.4	Operators	14
3.5	States	15
3.6	Properties	16
3.7	Specifying Tensors	16
3.8	Specifying Trajectories	18
3.8.1	Interaction trajectories	18
3.8.2	Tensor trajectories	19
3.8.3	Transition trajectories	19
3.8.4	Spin trajectories	19
3.9	ActionTargets: ActionVectors and ActionScalars	20
4	Specifying the settings object group	21
4.1	The Settings object	21
4.2	Actions	21
4.3	Output requests	22
5	The Run section and task classes	24
5.1	Overview of task classes	24
5.2	General parameters	25
5.3	Task class: DynamicHS-TimeEvolution	25
5.4	Task class: PeriodicHS-TimeEvolution	27
5.5	Task class: MultiDynamicHS-TimeEvolution	28
5.6	Task class: StaticSS-TimeEvolution	28
5.7	Task class: PeriodicSS-TimeEvolution	29
5.8	Task class: MultiStaticSS-TimeEvolution	30
5.9	Task class: StaticSS	31
5.10	Task class: StaticHS-SymmetricDecay	32
5.11	Task class: RP-SymmetricUncoupled	33
5.12	Task class: Gamma-Compute	34
5.13	Task class: TaskStaticSS-Redfield	34
5.14	Task class: TaskStaticSS-Redfield-TimeEvolution	41
5.15	Task class: MultiStaticSS-Redfield-TimeEvolution	44
5.16	Task class: RP-SymmetricUncoupled-Redfield	47

5.17	Task class: HamiltonianEigenvalues	50
5.18	Task class: StaticHS-StochYields	51
5.19	Task class: StaticHS-StochTimeEvo	54
5.20	Task class: StaticHS-DirectYields	56
5.21	Task class: StaticHS-DirectTimeEvo	58
5.22	Task class: DynamicHS-StochYields	59
5.23	Task class: DynamicHS-StochTimeEvo	62
5.24	Task class: DynamicHS-DirectYields	63
5.25	Task class: DynamicHS-DirectTimeEvo	65
5.26	Task class: StaticSS-PumpProbe	67
6	Preprocessor directives	71
6.1	#include <filename>	71
6.2	#define <name> <value (optional)>	71
6.3	#undef <name>	71
6.4	#ifdef <name>	71
6.5	#ifndef <name>	71
6.6	#else	72
6.7	#endif	72

1 Installation

1.1 Linux

This software package relies on the Armadillo C++ Linear Algebra Library, which needs to be installed on the system prior to compilation of the MolSpin application. Furthermore, it is suggested that OpenBLAS and LAPACK are also installed on the system, and you should make sure that they are recognized by Armadillo when installing that package (when you install Armadillo it will tell you whether it has detected those packages).

You can use other high-speed math libraries like Intel MKL instead of OpenBLAS, but you may need to modify the makefile in that case. Note also if you are *not* using OpenBLAS that you need to change *main.cpp*: import and usage of the function “openblas_set_num_threads” should be removed (it is just 2 lines that should be removed).

To compile the MolSpin application, navigate to the folder containing *makefile* and *main.cpp* in a terminal, and type **make**. If everything works, you will then have the executable file called “molspin” in the folder. If it does not work, you may need to change the makefile or configure your Armadillo installation.

Note that MolSpin was developed using the GCC 5.4.0 compiler and requires a C++14-compatible compiler. The Armadillo version should be 8.5 or newer.

Note also that installation (or rather compilation) of MolSpin does not generate any files outside the MolSpin folder containing the source code.

1.2 Windows

In Windows it would be easiest to use the Windows Subsystem for Linux (WSL) or Cygwin for compiling and running MolSpin. I have not tried compiling MolSpin on Windows outside WSL/Cygwin.

1.3 Testing your MolSpin installation

Once MolSpin is installed, you should check that everything works by running **make test**.

2 Getting Started

2.1 Input file basics

To run MolSpin you must first prepare an input file which describes the spin system(s) you want to investigate, settings for your calculations, and the type of calculations you want to perform. In order to make this as easy an intuitive as possible, MolSpin uses a very elaborate format for input files, which in some ways look similar to the C programming language - but of course much simpler. Employing such an elaborate input format has the effect that input files may become rather large, and therefore you should know that it is possible to split the input into multiple files using the `#include` preprocessor directive described in section 6.1¹.

The following subsections walk you through the main elements of the input file formats.

2.1.1 Overall structure: Objects and Object Groups

All input in MolSpin is organized in *objects*, and each object must belong to an *object group*. The same syntax is used for defining objects and object groups: first a keyword and a name is specified, separated by a space, and then the contents of the object or object group is contained between `{` and `}`. The keyword denotes the type of object or object group, and the name is used to refer to it from other parts of the input file, or from the output or error messages from MolSpin. Here is an example where a *SpinSystem* object group is created, containing a *Spin* object.

```
1 SpinSystem MySystem
2 {
3   Spin SomeParticle
4   {
5     spin = 1/2;
6     type = electron;
7   }
8 }
```

Indentation (spaces and tabs) and line breaks are ignored by MolSpin, so the same definitions could be written as:

```
1 SpinSystem MySystem{Spin SomeParticle{spin=1/2;type=electron;}}
```

Although this way of writing the input is valid, it is not very easy to see what is going on, so it is recommended to format your input files like the first example above!

There are only three types of object groups: **SpinSystem**, **Settings** and **Run**, and note that the **Run** group, which is also referred to as the **run section** is special since it cannot be assigned a name². There is no limit to the number of objects that can be assigned to an object group, and it is possible to split an object group between multiple files by simply creating object groups of the same type (e.g. SpinSystem) using the same name.

Different object groups contain different types of objects: SpinSystem groups contain objects of the types Spin, Interaction, Transition, State and a single Properties object. Settings groups contains a Settings object and any number of Action and Output objects, and the Run section contains the Task objects that describe what kinds of calculations to perform. Sections 3, 4 and 5 describes all the details about how these objects work, and you should be aware that Action objects are crucial for defining the behaviour of multi-step calculations, and is the main source of the great flexibility that MolSpin offers.

¹Note also that you can use `#define` directive to affect parameters defined in included files.

²All objects and both of the other object groups must be given a name; the run section is the only exception.

2.1.2 Comments

The MolSpin input format uses the same conventions for comments as in the C programming language: two slashes (//) are used for single-line comments, such that the rest of the line after // is considered a comment, and /* */ for multi-line or in-line comments. Here is example:

```
1 This is not a comment // But this IS a comment!
2 Not a comment /* this is a comment */ Not a comment again!
3 /*
4 Here is a comment
5 that span multiple lines!
6 */
```

2.1.3 Case sensitivity and strings

The input format is **not case sensitive** for normal input, but there is one notable **exception: strings** are case sensitive.

Strings are defined between quotation marks, and have the special property that they preserve spaces. Thus whenever you need to use capital letters, e.g. in a file name, or when you need to preserve spaces, e.g. when defining a vector or matrix, you need to use strings. Here is an example where the magnetic field vector of the Zeeman interaction is defined using a string:

```
1 Interaction Zeeman
2 {
3   type = Zeeman;
4   field = "0 0 1e-5";
5   spins = electron1, electron2;
6 }
```

One should in general be very careful when using strings due to the case sensitivity they imply. Especially when writing names; the names of objects or object groups can become case sensitive by using a string:

```
1 Interaction "Zeeman" // The name is now case sensitive - only strings can refer to it!
2 { /* contents... */ }
```

2.1.4 Input file example

As an example we will setup a calculation of the singlet yield of a radical pair for different external magnetic fields. Let us first create a SpinSystem with the two unpaired electronic spins of the radical pair:

```
1 SpinSystem RPSystem
2 {
3   Spin electron1
4   {
5     spin = 1/2;
6     type = electron;
7     tensor = isotropic(2);
8   }
9
10  Spin electron2
11  {
12    spin = 1/2;
```

```

13     type = electron;
14     tensor = isotropic(2);
15 }
16
17 // Other objects will be added here
18 }

```

This defines two spin-1/2 particles and sets the Landé g-factor to 2 (i.e. the isotropic g-tensors). Let us assume that each radical has a single magnetic nucleus, so we add two more spins to the SpinSystem:

```

1 Spin nucleus1
2 {
3     spin = 1/2; // This is a spin-1/2 like a proton
4     type = nucleus;
5     tensor = isotropic("1.0");
6 }
7
8 Spin nucleus2
9 {
10    spin = 1; // This is a spin-1 nucleus like nitrogen
11    type = nucleus;
12    tensor = anisotropic("0.5 0.5 0");
13 }

```

We will use the tensors of the nuclei as hyperfine tensors, and we want the specified tensors to be in mT units. Since the units by default are T, we can set the “prefactor” parameter of the hyperfine interaction objects to 10^{-3} . Thus the hyperfine interaction objects are:

```

1 Interaction Hyperfine1
2 {
3     type = Hyperfine;
4     group1 = electron1;
5     group2 = nucleus1;
6     prefactor = 1e-3;
7 }
8
9 Interaction Hyperfine2
10 {
11    type = Hyperfine;
12    group1 = electron2;
13    group2 = nucleus2;
14    prefactor = 1e-3;
15 }

```

Next we will add the external magnetic field as a Zeeman interaction, but only with the electrons; the interaction between the magnetic field and the nuclei is so small that it will be neglected here³:

```

1 Interaction Zeeman
2 {
3     type = Zeeman;
4     field = "0 0 5e-5";
5     spins = electron1, electron2;
6 }

```

³The difference is in the gyromagnetic ratio of the nuclei. If you want to add a Zeeman interaction with the nuclei you should do it as a separate Interaction object with “CommonPrefactor” set to false and by setting the prefactor of the Interaction object to the gyromagnetic ratio.

Next, the reaction processes and initial state of the radical pair should be specified, but in order to this you first need to define some State objects (also within the SpinSystem). In these calculations we are only concerned with the spin state of the two unpaired electrons, so we can just ignore the nuclear spins when defining the State objects. We will define a set of states that makes up a complete basis of the Hilbert space spanned by the two electronic spins:

$$|S\rangle = \frac{1}{\sqrt{2}}(|\alpha\beta\rangle - |\beta\alpha\rangle), \quad |T_+\rangle = |\alpha\alpha\rangle,$$

$$|T_0\rangle = \frac{1}{\sqrt{2}}(|\alpha\beta\rangle + |\beta\alpha\rangle), \quad |T_-\rangle = |\beta\beta\rangle,$$

where α and β are the two spin states of an electronic spin. In MolSpin these states are defined as:

```

1  State Singlet
2  {
3    spins(electron1,electron2) = |1/2,-1/2> - |-1/2,1/2>;
4  }
5
6  State T0
7  {
8    spins(electron1,electron2) = |1/2,-1/2> + |-1/2,1/2>;
9  }
10
11 State Tp
12 {
13   spin(electron1) = |1/2>;
14   spin(electron2) = |1/2>;
15 }
16
17 State Tm
18 {
19   spin(electron1) = |-1/2>;
20   spin(electron2) = |-1/2>;
21 }
22
23 State Identity
24 {
25 }
```

The normalization constants of $\frac{1}{\sqrt{2}}$ are not necessary as MolSpin will normalize the states for you. Note that we have defined an additional state object without any contents, the *Identity* state, which will serve a special purpose: now that we are ready to define a decay process for the radical pair, the Identity state - which does not specify any particular state for any spin - can be used to specify spin-independent processes. Let us therefore just define a single, spin-independent decay process with a rate constant of $1 \mu\text{s}^{-1}$:

```

1  Transition spinindependent_decay
2  {
3    rate = 1e-3;           // The rate constant
4    source = Identity;    // Specify the state here
5  }
```

Note that if you wanted to have triplet decay from your radical pair, you would need to create three Transition objects: one for each of the states $|T_0\rangle$, $|T_+\rangle$ and $|T_-\rangle$ defined above; it is not possible to define a single generic triplet state. Now we only need to specify the initial state, which is done using a special *Properties* object:


```

1 Properties properties
2 {
3   initialstate = Singlet;
4 }

```

Currently no other properties can be specified for a spin system. If you instead want your system to be spawned not in a particular state like $|T_0\rangle$, $|T_+\rangle$ or $|T_-\rangle$, but a mixed ensemble containing an equal amount of all three of these, you can specify a comma-separated list of states to be represented initially:

```

1 Properties properties
2 {
3   initialstate = T0, Tp, Tm; // Mixed triplet ensemble
4 }

```

Note that some calculation methods may not describe ensembles, but the evolution of a specific state, in which case the above initial state specification will create a linear combination of $|T_0\rangle$, $|T_+\rangle$ or $|T_-\rangle$ as the initial state (i.e. just a single state!). No such calculation methods are implemented at the moment, but it may be the case for some custom methods created by other users.

That completes our description of the SpinSystem - the physical system is now described! Next we will create the so-called *Run section* of the input file which is just a collection of *Tasks*, i.e. the actual calculations to be carried out. We only want to perform a simple quantum yield calculation, so we will use the following Run section:

```

1 Run
2 {
3   Task CalculateQuantumYield
4   {
5     type = RP-SymmetricUncoupled;
6     logfile = "logfile.txt"; // The log will be written to this file
7     datafile = "result.dat"; // Data will be written to this file
8   }
9 }

```

The Run section is not assigned a name, and we define a single task of the type *RP-SymmetricUncoupled* which is a very special task type that can only be used under special circumstances - but which is very fast. It is recommended to read more about the main task types: **StaticSS**, **DynamicHS-TimeEvolution**, **StaticHS-SymmetricDecay**, and **RP-SymmetricUncoupled**. All of these can calculate quantum yields, while **StaticSS-TimeEvolution** and **DynamicHS-TimeEvolution** are the main task types for calculating the time evolution of a spin system. If you have any time-dependent interactions you should normally go with DynamicHS-TimeEvolution, but this task type is very slow.

Now we just need to specify the settings for the calculation, in particular we need to specify Actions, Outputs and a Settings object, and these are specified in the Settings object group:

```

1 Settings
2 {
3   Settings general
4   {
5     steps = 500;
6   }
7
8   Action scan
9   {

```

```

10     value = 0.001;
11     direction = "0 0 1";
12     type = addvector;
13     vector = RPSystem.Zeeman.field;
14 }
15
16 Output fieldstrength
17 {
18     type = length;
19     vector = RPSystem.Zeeman.field;
20 }
21 }

```

We specify that we want to perform 500 steps, that is run all the tasks in the run section 500 times. Since we do not just want to repeat the exact same calculation 500 times, we also define an Action, which adds a vector along the z -axis with a length of 1 mT to the external magnetic field between every step (i.e. the action is executed every time all of the tasks in the Run section has finished). Finally we also create an Output object, telling MolSpin that we would like to see the current field strength every time results are written. Note that use of such Output objects are not mandatory for the task classes, so not every task type may make use of it - but most task types will.

You are now ready to run MolSpin using your input file:

```

1 molspin my_input.msdx

```

The **.msdx** file extension is recommended for MolSpin input files. You can read more about running MolSpin in the next section.

2.2 Running MolSpin

MolSpin is run from the commandline using the following syntax:

```

1 <path to molspin executable> <parameters> <path to input file>

```

If the folder containing the MolSpin executable file is in your PATH environment variable you can simply write:

```

1 molspin <parameters> <path to input file>

```

If your input file is called "my_input.msdx" (the **.msdx** extension is recommended for MolSpin input files, but not required), then you would write:

```

1 molspin <parameters> my_input.msdx

```

This assumes that you are currently in the folder containing the input file – otherwise you would need to specify the path to the file. Note also that you can specify some commandline parameters, and that these *must be given before the input file*.

2.2.1 Commandline parameters

The available commandline parameters are listed below:

Parameter	Example	Description
-a --append	molspin -a myfile.msdc	File output is appended to existing files (instead of overwriting).
-c --checkpoint	molspin -c mytask myfile.msdc	Skip all tasks in the runsection until the specified checkpoint is found. This only happens for the first step.
-d --defines	molspin -d myfile.msdc molspin -d def 5 myfile.msdc	Show all defined directives and their values, and included files. Can also be used to create defines on the commandline by specifying name and value to define (beware of case sensitivity).
-h --help	molspin -h	Shows help message if no input file is specified.
-n --steps	molspin -n 5 myfile.msdc	Specify how many steps should be calculated before you are notified.
-o --objects	molspin -o myfile.msdc	Show the objects that were read from the input files.
-os --objects-states	molspin -os myfile.msdc	Similar to --objects, but with extra State object information.
-p --threads	molspin -p 24 myfile.msdc	Specify the number of threads/processor cores to use. Works only if you are using OpenBLAS.
-r --first-step	molspin -r 5 myfile.msdc	Specify which step to start from (if you don't want to start from step 0).
-l --step-limit	molspin -l 5 myfile.msdc	Limits the number of steps to run.
-s --silent	molspin -s myfile.msdc	Minimize the text output.
-t --action-targets	molspin -t myfile.msdc	Prints all the ActionTargets that can be used.
-z --no-calc	molspin -z myfile.msdc	Skip calculations.

Note that you can restart a stopped calculation using:

```
1 molspin -r <step number> -c <task name> -a myfile.msdc
```

Here you can specify the last unfinished step number, and the task name of the first unfinished task that appears in the run section. If all tasks were completed for the last step number that was run (or if you have only a single task in the run section), and you merely wish to extend the calculation with more steps, you can omit the checkpoint specification. Make sure that you are using the append-option though, as you will otherwise overwrite the existing files!

3 Specifying the spin system

A *SpinSystem* defines a physical system. It is an object group that contains Spins, Interactions, Transitions, States and a Properties object, and it is possible to divide a SpinSystem definition between multiple files using the `#include` directive. SpinSystems are defined as follows:

```
1 SpinSystem my_system
2 {
3   // Define objects (Spins, Interactions, etc.) here...
4 }
```

It is possible to define multiple spin systems, but note that not all task types supports interactions between spin systems.

3.1 Spins

Spins are defined by the keyword *spin* followed by a name:

```
1 Spin Nitrogen_nucleus
2 {
3   spin = 1; // 14N is a spin-1 particle
4   tensor = isotropic(1) + anisotropic("0 0 -2.0");
5   type = nucleus;
6 }
7 Spin electron
8 {
9   spin = 1/2;
10  tensor = isotropic(2); // The anomalous magnetic moment of the electron (without QED)
11  type = electron;
12 }
```

The following parameters can be defined for a spin:

Parameter	Values	Default	Description
Spin	<half-integer>	1/2	The spin quantum number in units of \hbar .
Tensor	<tensor>	isotropic(2)	The g -tensor.
Type	<i>electron</i> , <i>nucleus</i> , <i>unspecified</i>	<i>unspecified</i>	What type of spin is it (i.e. electron, atomic nucleus or something else?).
QuantizationAxis1	<vector>	"1 0 0"	First spin quantization axis.
QuantizationAxis2	<vector>	"0 1 0"	Second spin quantization axis.
QuantizationAxis3	<vector>	"0 0 1"	Third spin quantization axis.

Note that the spin quantum number is given as a “half-integer”: You must specify either an integer or an integer with the suffix “/2”. The allowed values of the spin quantum number are therefore: 1/2, 1, 3/2, 2, 5/2, 3, and so on. Note that you can also write e.g. 2/2 instead of 1, but decimal numbers such as 0.5 are not allowed.

The spin quantization axes determine the spin operators in the lab frame, i.e. $\mathbf{S}'_x = a_1\mathbf{S}_x + a_2\mathbf{S}_y + a_3\mathbf{S}_z$ where a_1 , a_2 and a_3 are the components of the first quantization axis. Note that the lab frame spin operators are, furthermore, affected by the tensor as well, and in principle the quantization axis specification is redundant as the same results could be obtained using a rotation matrix for the tensor - but sometimes it is just easier to set the quantization axes than calculating the needed transformation matrix.

3.2 Interactions

All interactions come in one of the following two forms:

$$\mathbf{H}_{\text{SS}}(i) = \mu_B \alpha \mathbf{g}_i \mathbf{S}_i \cdot \mathbf{A} \cdot \mathbf{F}, \quad \mathbf{H}_{\text{DS}}(i, j) = \mu_B \alpha \mathbf{g}_i \mathbf{S}_i \cdot \mathbf{A} \cdot \mathbf{g}_j \mathbf{S}_j, \quad (1)$$

where i and j are indices referring to two spins. μ_B the Bohr magneton, \mathbf{S}_k are spin operators, and \mathbf{g}_k are the tensors specified in the spin objects. \mathbf{A} , \mathbf{F} and α is a tensor, a vector and a constant factor, respectively, specified for the Interaction object. The Hamiltonian \mathbf{H}_{SS} involves only a single spin (this could be the Zeeman interaction where \mathbf{F} would be a magnetic field), while \mathbf{H}_{DS} will be referred to as the double-spin Hamiltonian (e.g. hyperfine or exchange interaction).

The following parameters can be specified for an Interaction:

Parameter	Values	Default	Description
Type	<i>singlespin</i> , <i>doublespin</i>	-	One of the two types of interaction in Eq. (1). Must always be specified.
Tensor	<tensor>	isotropic(1)	The tensor \mathbf{A} used in Eq. (1).
Field	<vector>	zero- vector	The vector \mathbf{F} used in Eq. (1). Note: Only used in single-spin interactions!
Prefactor	<float>	1.0	Optional prefactor for the interaction object. α in Eq. (1).
CommonPrefactor	<bool>	true	If set to false, the Bohr magneton μ_B will not be included in Eq. (1) for the interaction object.
IgnoreTensors	<bool>	false	If set to true, the tensors of the spin objects \mathbf{g}_i and \mathbf{g}_j will not be included in Eq. (1) for the interaction object.
FieldType TimeDependence	<i>LinearPolarization</i> , <i>CircularPolarization</i> , <i>Static</i>	<i>Static</i>	Allows the Field vector of the interaction object to become time-dependent. Note: Not all task types support time-dependent Hamiltonians!
Frequency	<float>	1.0	Frequency of field oscillations. Requires a FieldType other than <i>Static</i> .
Phase	<float>	0.0	Initial phase of field oscillations. Requires a FieldType other than <i>Static</i> .
Axis	<vector>	"0 0 1"	Axis that the field will rotate about. Requires FieldType set to <i>CircularPolarization</i> .
Perpendicular- Oscillations	<bool>	false	If set to true, the rotating field vector will be projected onto the plane perpendicular to the rotation axis. Requires FieldType set to <i>CircularPolarization</i> .

Note that there are some synonyms for the *Type* parameter: you can write *onespin* or *Zeeman* instead of *singlespin*, and you can use *twospin*, *hyperfine*, *dipole* or *exchange* instead of *doublespin*.

3.3 Transitions

A transition object describes a process that can change the total number of spin systems, i.e. it represents a source or a sink. Transition objects are commonly used to model decay or recombination reactions (sinks) or processes that replenishes the spin system. For transitions of the sink type it is also possible to specify a target spin system together with a target state, meaning that the transition/process transforms the spin system into another spin system - an example of such a process is an

electron transfer from a radical, that generates another radical (since both radicals are spin systems, but with different molecular environment). Note that target spin systems are not taken into account by most task types; see the individual task types for more information.

Transition objects have the following parameters:

Keyword	Values	Description
Type	<i>source</i> , <i>sink</i>	Determines whether the process is a sink (default) or a source.
Source State SourceState	<State>	The state in the SpinSystem which is diminished or augmented.
Target TargetSystem	<SpinSystem>	The new SpinSystem that the transition leads to. Requires specification of a target state, and transition type must be <i>sink</i> .
TargetState	<State>	A state in the target SpinSystem.
ReactionOperators ReactionOperatorType	<i>Haberkorn</i> , <i>Lindblad</i>	Requests a specific type of reaction operator for this transition (Haberkorn is default).
Rate	<float>	The rate constant for the transition in ns ⁻¹ .
Lifetime	<float>	The inverse of the rate constant for the transition in ns. Only used if rate is not specified (either directly or in a trajectory).

Note that the reaction operator type is not taken into account by all task classes.

3.4 Operators

Operators are may be used for a variety of purposes by custom task classes, but their original intended purpose is to describe relaxation operators. Operator objects are a recent addition to the SpinAPI, and their uses in the implemented task classes is limited.

Operator objects have the following parameters:

Keyword	Values	Description
Type	<i>RelaxationLindblad</i> <i>relaxationdephasing</i> <i>unspecified</i>	The type of operator.
Rate1	<float>	A value that may be associated with the operator.
Rate2	<float>	A value that may be associated with the operator.
Rate3	<float>	A value that may be associated with the operator.
Rates	<float>	Sets all three rates to this value.
Spins SpinList	<Spins>	Spin objects that should be affected by the operator.

Note that the SpinAPI::SpinSpace class only supports the Operator type *RelaxationLindblad*, and

only for Liouville-space task classes such as *StaticSS*. The produced relaxation is defined by:

$$\mathbf{R}(\rho) = \sum_i \left[k_x \left(\mathbf{S}_{ix} \rho \mathbf{S}_{ix} - \frac{1}{2} (\mathbf{S}_{ix} \rho + \rho \mathbf{S}_{ix}) \right) + k_y \left(\mathbf{S}_{iy} \rho \mathbf{S}_{iy} - \frac{1}{2} (\mathbf{S}_{iy} \rho + \rho \mathbf{S}_{iy}) \right) + k_z \left(\mathbf{S}_{iz} \rho \mathbf{S}_{iz} - \frac{1}{2} (\mathbf{S}_{iz} \rho + \rho \mathbf{S}_{iz}) \right) \right],$$

where the sum is over all spins in the specified *spinlist*, and k_x , k_y and k_z are the three rates.

3.5 States

State objects define the quantum state of a spin system, although they often describe only the quantum state of a subsystem, for example the singlet state of a radical pair. There are only two types of input in a State object:

Keyword	Values	Description
spin(<spin>)	<state ket>	Specify the the state of a single spin.
spins(<spinlist>)	<state ket>	Specify the the state of entangled spins.

The first keyword, *spin*, is used to specify the spin state of a single Spin object inside the spin system. The spin state should be written as a state ket, i.e. $|1/2\rangle$ or $|-1/2\rangle$ for a spin-1/2 particle, or $|+1\rangle$, $|0\rangle$ or $|-1\rangle$ for a spin-1 particle. Note that the name of the Spin object is given between (and) as part of the keyword, and you can thus set the state for each spin individually as in the following example:

```

1 // Define some spin objects
2 Spin s1 { spin = 1/2; }
3 Spin s2 { spin = 3/2; }
4
5 // Define a quantum state
6 State MyState
7 {
8   spin(s1) = |1/2>;
9   spin(s2) = |-3/2>;
10 }
```

You do not need to specify a state for every spin in the spin system. Whenever the state object is used to generate a projection operator onto a quantum state (as is the case in almost all task classes), the resulting state projection operator will be an identity operator in the subspace spanned by the spins which have not been specified in the State object. As a special result of this, the projection operator corresponding to an empty State object is just the Identity operator for the spin system, and using such a State object as the source state in a Transition will lead to a spin-independent transition:

```

1 // Empty State object
2 State IdentityOperator { }
```

All task types that rely on a density operator formalism, for example, only use State objects to create projection operators (the initial ensemble described by a density operator can be written as $\rho(0) = \sum_i \mathbf{P}_i / \text{Tr}(\sum_i \mathbf{P}_i)$ where \mathbf{P}_i are projection operators). You need to be more careful with task types that use state vectors (e.g. when solving the Schrödinger equation) since state vectors need to represent a specific state - in such cases it is not possible to use a mixed ensemble of states. Such task types are not currently implemented in MolSpin though.

Specifying the state of each spin individually is not always enough, since spins can be entangled. Examples are the singlet and T_0 states of a radical pair. Such entanglements can be specified using the keyword *spins* by providing a comma-separated list of Spin object names between (and) as part of the keyword. The value is again written as a state ket, where the magnetic quantum number of each spin is given as a comma-separated list. Also, in order to properly specify entanglements, a linear combination of state kets can be specified:

```

1 // Define some spin objects
2 Spin s1 { spin = 1/2; }
3 Spin s2 { spin = 3/2; }
4 Spin s3 { spin = 1/2; }
5
6 // Define a quantum state
7 State MyEntangledState
8 {
9   spins(s1,s2) = 2|+1/2,-3/2> - 3i|+1/2,-1/2> + 0.1 |-1/2,3/2>;
10  spin(s3) = |1/2>;
11 }

```

Note that MolSpin does not expect the specified state to be normalized; it is the responsibility of the task classes to normalize any state vectors or spin state ensembles.

3.6 Properties

A spin system can have a single Properties object which defines some general properties for the system. The following properties are available:

Keyword	Values	Description
InitialState	<State list>	A comma-separated list of states describing the initial state of the spin system.

If multiple initial states are specified, note that the exact meaning of having multiple initial states depends on the used task types (especially if custom task types are used). All of the standard task types described in this manual treats a spin system as an ensemble which initially contains an even mixture of all the specified states, unless otherwise specified in the task class description.

3.7 Specifying Tensors

Tensor objects can be specified for Interaction and Spin objects, and play the role of hyperfine and g-tensors. Since tensors are complex entities, a special syntax must be used to specify them, and this syntax consists of a combination of the following keywords:

Keyword	Values	Description
Isotropic	<float>	The isotropic value of the tensor.
Anisotropic	<vector>	The three components of the anisotropic part.
Matrix	<matrix>	A full matrix representation of the tensor. The matrix is expected to be symmetric.
axis1 axis2 axis3	<vector>	The principal axes of the tensor.
ChangeBasis	<matrix>	A similarity transformation will be applied to the lab frame matrix representation of the tensor, i.e. $\mathbf{A}' = \mathbf{S}^\dagger \mathbf{A} \mathbf{S}$ where \mathbf{A} is the current lab frame representation, \mathbf{A}' is the lab frame representation after the transformation, and \mathbf{S} is the transformation matrix specified by this keyword.
Trajectory	<filename>	Load a trajectory for the tensor, and use it to set the tensor parameters.

Every keyword is followed by parentheses containing the parameters, and multiple keywords are added by a plus sign. For example:

```

1 tensor = isotropic("1.0") + anisotropic("1e-4 1e-4 1e-3") + trajectory("motions.mst");
2 tensor = matrix("1.0 0.0 0.0;0.0 1.0 0.0;0.0 0.0 1.0");

```

The trajectory in the first example could for example contain columns with time and the three principal axes, e.g. describing rotational diffusion of a radical while assuming constant principal values (isotropic and anisotropic values) of the tensor. Note that the matrix in the second example above is an identity matrix, so the same definition could be made using **isotropic("1.0")** instead. The examples above also demonstrate **how vectors and matrices are defined**: vectors consists of three numbers separated by **spaces**, and must be written as **strings**⁴, and matrices are defined in a similar way, with **semicolon** to separate rows in the matrix.

Important note on tensor definitions: The order in which components of a tensor are added together matters! The keywords for a tensor are processed in the order in which they appear; in the first example above, the isotropic part is first set, then the anisotropic part, and then finally a trajectory is assigned. There is no conflict in any of these examples, but consider instead the following two cases which yield very different results:

```

1 tensor = axis1("1 1 0") + axis2("1 -1 0") + matrix("1 0.1 -0.3;0.1 1 0.7;-0.3 0.7 -2");
2 tensor = matrix("1 0.1 -0.3;0.1 1 0.7;-0.3 0.7 -2") + axis1("1 1 0") + axis2("1 -1 0");

```

The **matrix** keyword diagonalizes the given matrix and sets the three principal axes to the eigenvectors of the matrix, regardless of what the axes were assigned to previously. Hence the first line above is *wrong*: the axis specifications are **overwritten** by the *matrix* keyword! Note also that any transformation performed by the *changebasis* keyword will be overwritten by the axis keywords. Thus **it is recommended** that you *use the axis keywords after the matrix keyword (as in the second line above), and the changebasis keyword after the axis keywords.*

⁴In order to preserve the spaces that would otherwise be ignored by the input file parser!

3.8 Specifying Trajectories

Trajectories is a special feature in MolSpin that allows you to specify complex time-dependences of parameters. For some task types they may also be used to provide sets of parameters, without specifying any time - such uncommon uses will be mentioned explicitly in the documentation for the individual task types. A trajectories are specified in separate files which contain only a trajectory, and it is recommended to use the `.mst` file extension to easily identify trajectory files (**MolSpin Trajectory**).

A trajectory file can be perceived as a table consisting of some rows and columns. The first line of the trajectory file defines the column headers, and each following line defines a row with data. Each column is separated by one or more space or tab characters; this holds for both the header row and all the data rows.

Below is an example of a trajectory file for an Interaction object:

```
1 time prefactor field.x field.y field.z
2 100 1 0 0 5e-4
3 300 1 5e-4 0 0
4 400 1 0 5e-4 0
5 600 5 0 0 5e-4
6 800 0.1 0 0 0.001
```

The **time** column has a special meaning and is required for task types that includes time-dependent parameters: when the parameters in the trajectory are requested at a specific time (here the prefactor and field of an Interaction object), *the first entry in the trajectory* is used for all times *smaller* than the first specified time. When the requested time is *larger* than any entries in the trajectory it is instead *the final entry in the trajectory* that is used. For all requested times between that of the first and the final entry in the trajectory, the first entry with a time larger than the requested time is found, and a **linear interpolation** between this and the previous row in the trajectory is used for all the parameters. It is currently not possible to change the interpolation type, but since there is no limit on the number of rows in the trajectory you still have fine-grained control⁵. Note that MolSpin expects the rows to be **ordered by time**.

3.8.1 Interaction trajectories

Interaction objects recognize the following column headers:

time, prefactor, field.x, field.y, field.z.

Note that if any of the field components (field.x/y/z) are used, it is required that the remaining components are specified as well. Otherwise the specified field components are ignored (i.e. you must either specify both field.x, field.y and field.z, or none of the three).

Trajectories for the tensor of an interaction is specified separately by adding the *trajectory* keyword to the tensor specification, and can be a separate file. You can use the same trajectory file for both the Interaction object and its tensor since Interaction and Tensor objects use different column header names (except time) and each object type will just ignore the columns that it does not use. Note that this approach still means that the trajectory file will be loaded into memory twice, and that there is a certain memory overhead (waste) with this approach - for very long trajectories it is therefore recommended to split the Interaction trajectory (prefactor and field.x/y/z) and its Tensor trajectory into two separate files.

⁵Memory limitations still apply; every trajectory is stored in the memory while MolSpin is running.

3.8.2 Tensor trajectories

Tensor objects recognize the following column headers:

time, isotropic, anisotropic.x, anisotropic.y, anisotropic.z, axis1.x, axis1.y, axis1.z, axis2.x, axis2.y, axis2.z, axis3.x, axis3.y, axis3.z

Note that if *one* component of a vector is specified, *all* components of the same vector must be specified (as with the field.x/y/z for Interaction objects). Thus if you want to specify the anisotropic part you must specify both anisotropic.x, anisotropic.y and anisotropic.z. Similar with e.g. axis2.x/y/z. But you do not need to specify all axes if you specify one of them; you can for example specify axis1 and axis3 without specifying axis2, as long as you just include both the x, y and z components of both axis1 and axis3.

Here is an example of a tensor trajectory:

```
1 time isotropic anisotropic.x anisotropic.y anisotropic.z
2 100 0 0.0 0.0 0.0
3 200 2 0.2 0.2 0.0
4 400 1 0.0 1.2 0.0
5 500 0 0.8 0.0 0.0
6 800 1 0.0 0.0 2.0
```

3.8.3 Transition trajectories

Transition objects can have trajectory that specifies the time-evolution of the rate constant, or alternatively the time-evolution can be specified for the lifetime instead of the rate constant (the lifetime is the inverse of the rate constant).

Here is an example of a Transition trajectory:

```
1 time rate lifetime
2 100 1e-2 100
3 200 1e-4 10000
4 400 0.0 1e+10
5 500 1e-3 1000
6 600 2e-3 500
```

Note: You should specify either a rate column or a lifetime column, since only one of them can be used (the lifetime also provides a rate). If both columns are present as in the example above, only the column with the “rate” header will be used.

3.8.4 Spin trajectories

For Spin objects their tensors can be assigned a trajectory, but the Spin object itself may also be assigned a trajectory with the spin quantization axes.

time, axis1.x, axis1.y, axis1.z, axis2.x, axis2.y, axis2.z, axis3.x, axis3.y, axis3.z

Note that the same trajectory file can be assigned to Spin objects and their tensor, since the Spin object will just ignore the additional data columns used by the tensor.

3.9 ActionTargets: ActionVectors and ActionScalars

Many parameters can be changed between the executing of calculation steps using *Actions* (see section 4.2), which provides a great deal of flexibility. These parameters that can be affected by Actions are collectively known as ActionTargets, and may be divided into ActionVectors and ActionScalars. An example of an ActionVector is the field vector of a singlespin Interaction, whereas the rate constant for a Transition object, for example, is an ActionScalar. In addition to their use in Actions, ActionTargets can also be used by Output objects (see section 4.3).

The name of an ActionTarget will be of the form “<SpinSystem name>.<object name>.<parameter>”, and you can use the “-t” commandline option to see a list of available ActionTargets for the spin system(s) you have defined:

```
1 molspin -t -z myfile.ms
```

Here the “-z” option is also used to prevent running any calculations, assuming you only wanted to see a list of ActionTargets.

Note: Make sure that all your objects within a spin system have unique names, and that your spin systems have unique names, such that each ActionTarget can be uniquely identified.

4 Specifying the settings object group

The Settings object group allows you to define *Actions* and *Output requests*, as well as some general options for your calculations.

4.1 The Settings object

The Settings object should contain a Settings object, which is used to specify general options for the calculations. The following settings are available:

Keyword	Values	Description
Steps	<integer>	The number of calculation steps to perform, i.e. number of times the Run section is being executed. Default: 1.
Time	<float>	Sets the time for any time-dependent interactions or trajectories. Only useful for task classes that do not support time-dependent interactions (in order to “freeze” time-dependent interactions at specific values for the calculations). Default: 0.
TrajectoryStep	<integer>	Sets the trajectory step for any parameter with a trajectory. For trajectories including a time column this is only useful for task classes that do not support time-dependent interactions. Default: 0.
TrajectoryStep- BeforeTime	<bool>	If true, the trajectory step will be set before the time, which means that trajectories with a time column will use time instead of trajectory step. Default: true, except when <i>TrajectoryStep</i> and not <i>Time</i> is specified.
Notifications	<i>quiet</i> <i>sparse</i> <i>normal</i> <i>details</i>	Specifies the notification level. Default: Normal.
IgnoreWarnings Nowarn	<bool>	Ignore warnings. Default: False.
IgnoreErrors Noerr	<bool>	Ignore error messages. Default: False.

Note that *TrajectoryStepBeforeTime* should be written without “-”. Note also that some task classes do not distinguish between notification levels for log output.

4.2 Actions

The use of *actions* is a central feature of the MolSpin application, as they provide a very general framework for setting up calculations. When you set up your SpinSystems you are able to perform a single calculation, but if you want to see the effect of changing a parameter on e.g. the quantum yields, multiple calculations are needed. Hence you can specify the number of calculations you want to be performed as the *steps* parameter in the settings object. But without *Actions*, all of those calculations will be identical – it is the role of an Action to change the parameters between these individual calculations.

The following types of actions are available, with the Action-specific parameters shown together with each Action Type:

Action Type	Parameter	Description
RotateVector		Rotates an ActionVector about a specified axis. Rotation angle is given as the action value in degrees.
	Vector ActionVector	The ActionVector to act on.
	Axis	A vector specifying the axis to rotate the ActionVector about.
ScaleVector		Scales an ActionVector by multiplication of the action value.
	Vector ActionVector	The ActionVector to act on.
AddVector		Adds a constant vector to the ActionVector.
	Vector ActionVector	The ActionVector to act on.
	Direction	A vector specifying the direction of the vector to be added to the ActionVector. The length of the vector is set to the action value.
MultiplyScalar ScaleScalar		Multiplies an ActionScalar by the constant action value.
	Scalar ActionScalar	The ActionScalar to act on.
AddScalar		Adds the constant action value to an ActionScalar.
	Scalar ActionScalar	The ActionScalar to act on.

In addition to the Action-type-specific parameters, the following list of parameters are available for all Actions:

Parameter	Values	Description
Type	<type>	A type of Action object from the table above.
Value	<float>	This is the “action value” referred to by the specific actions, and its function depends on the action type (see the action type table above).
Period	<integer>	Specifies how frequently the action should be performed. A period of 1 (default) means that the action will be performed after every step, a period of 5 means it will be performed after every 5th step.
First	<integer>	The first step at which the action should be performed (default 1). After this step, the action will be performed with a frequency specified by the action period.
Last	<integer>	The last step at which the action may be performed. After reaching this step, the action will not be performed anymore.

4.3 Output requests

In addition to the standard output from the various task classes, such as quantum yields, you can request additional output using *Output* objects. Note that Output objects only requests output, but it

is up to the individual task classes to decide whether when and where this requested output is printed, and task classes may choose to ignore it (but this is not the case for any of the currently implemented task classes).

Output requests are used to obtain information about ActionTargets: either by printing the actual ActionScalar value or ActionVector components, or by requesting information such as the length of an ActionVector. All the possible Output request types are listed in the table below:

Output Type	Parameter	Description
Angle VectorAngle		Outputs the angle (degrees) between two vectors.
	Vector	A pointer to the ActionVector used to calculate the angle.
	Reference	A constant vector used to calculate the angle.
Length VectorLength		Outputs the lengths of a vector.
	Vector	A pointer to the ActionVector whose length should be calculated.
Components XYZ Vector VectorXYZ		Outputs the Cartesian components of a vector.
	Vector	A pointer to the ActionVector whose components to write to the output file.
Dot VectorDot Projection		Calculates the projection (dot product) of the vector onto a reference vector.
	Vector	A pointer to the ActionVector used to calculate the projection.
	Reference	A constant vector the ActionVector is projected onto.
Scalar		Outputs a scalar
	Scalar	A pointer to the ActionScalar to write to the output file.

In addition, Output objects have the following properties that can be specified:

Keyword	Values	Description
Type	<type>	A type of output object from the table above.
Prefactor	<float>	Any value will be multiplied by this number before being printed.

5 The Run section and task classes

The Run section defines what happens at every calculation step: each task specified in the Run section are executed sequentially within a calculation step, and this execution is repeated for the number of calculation steps defined in a Settings objects (i.e. in the Settings object group, not the Run section which only defines what happens every step). The tasks in the Run section are executed in the order that they appear in the input file.

5.1 Overview of task classes

There is a wide variety of task classes available, as summarized in the table below. In general their names refer to Hilbert Space (HS) or Superoperator Space (SS) methods, where superoperator space (Liouville space) methods in general requires much more memory (i.e. scales much worse with number of spins). The prefixes *Static*, *Dynamic* and *Periodic* tells whether time-dependent interactions are supported. Note that some methods are specialized to specific spin systems; the RP-SymmetricUncoupled method is designed for radical pairs only.

Task class	Type	TD	RT	MS	Memory	Speed
<i>Liouville-Von Neumann Formalism</i>						
DynamicHS-TimeEvolution	TE/QY	Yes	All	No	Medium	Very slow
PeriodicHS-TimeEvolution	TE/QY	Per.	All	No	Medium	Slow
MultiDynamicHS-TimeEvolution	TE	Yes	All	Yes	Medium	Very slow
StaticSS-TimeEvolution	TE	No	All	No	High	Medium
PeriodicSS-TimeEvolution	TE	Per.	All	No	High	Medium
MultiStaticSS-TimeEvolution	TE	No	All	Yes	High	Medium
StaticSS	QY	No	All	No	High	Medium
StaticHS-SymmetricDecay	QY	No	Indep.	No	Medium	Fast
RP-SymmetricUncoupled	QY	No	Indep.	No	Low	Very fast
Gamma-Compute	QY	Per.	Indep.	No	Medium	Fast
StaticSS-Redfield	QY	Ind.	All	No	High	Slow
StaticSS-Redfield-TimeEvolution	TE	Ind.	All	No	High	Slow
MultiStaticSS-Redfield-TimeEvolution	TE	Ind.	All	Yes	High	Slow
RP-SymmetricUncoupled-Redfield	QY	Ind.	All	Yes	High	Slow
<i>Other</i>						
Eigenvalues	EL/RES	Yes	–	No	Medium	Fast
<i>Schrödinger Formalism</i>						
StaticHS-StochYields	QY	No	All	No	Low	Very Fast
StaticHS-StochTimeEvo	TE	No	All	No	Low	Very Fast
StaticHS-DirectYields	QY	No	All	No	Low	Fast
StaticHS-DirectTimeEvo	TE	No	All	No	Low	Fast
DynamicHS-StochYields	QY	No	All	No	Low	Very Fast
DynamicHS-StochTimeEvo	TE	No	All	No	Low	Very Fast
DynamicHS-DirectYields	QY	No	All	No	Low	Fast
DynamicHS-DirectTimeEvo	TE	No	All	No	Low	Fast

Type: Describes what can be calculated, which is either the time-evolution (TE), quantum yields (QY), energy levels (EL) or resonance frequencies (RES). **TD:** Whether time-dependences are taken into account, where *Per.* means only periodic time-dependences and *Ind.* indirect through Redfield theory. **RT:** Supported reaction types, where *Indep.* means that only a spin-independent reaction is supported. **MS:** Supports use of multiple spin systems. Most task types without multi-system support can still handle each spin system individually, but ignores transitions between spin systems. **Memory and speed:** The memory and calculation time requirements of a task class, which for most task types scales exponentially with system size.

5.2 General parameters

There are a few parameters that are available to all task classes; these are shown in the table below.

Parameter	Values	Description
Logfile Log Output	<filename>	Location of logfile. If no logfile is specified, log output will be written to stdout.
Datafile Data	<filename>	Location of main data file. If no data file is specified, main data output will be written to stdout.
AppendLog	<bool>	If set to true, log output will be appended to the specified logfile instead of overwriting an existing file. Not enabled by default.
AppendData	<bool>	If set to true, data output will be appended to the specified main data file instead of overwriting an existing file. Not enabled by default.
Append	<bool>	Setting this option overrides both AppendLog and AppendData. Note that the command-line append option also overrides this option.
Notifications	<i>quiet</i> <i>sparse</i> <i>normal</i> <i>details</i>	Specifies the notification level. Default: Normal.
IgnoreWarnings Nowarn	<bool>	Ignore warnings. Default: False.
IgnoreErrors Noerr	<bool>	Ignore error messages. Default: False.

5.3 Task class: DynamicHS-TimeEvolution

Brief description

Brute-force numerical integration to obtain the time-evolution of the spin system ensemble. Can perform time integration of these results in order to output quantum yields instead.

Limitations, approximations and comments

- This is the most general method to compute time-evolution or quantum yields for a single Spin-System; it can handle all types of input.

- MultiDynamicHS-TimeEvolution is the equivalent method to use when you need multiple SpinSystems.
- Make sure to use a small enough timestep to avoid significant numerical errors.
- When calculating quantum yields, make sure the totaltime parameter is long enough to let the SpinSystem decay completely.
- This is a very slow method; use it only when faster methods do not suffice.

Parameters

Parameter	Values	Default	Description
TimeStep	<float>	0.01	Numerical integration timestep (in ns).
TotalTime	<float>	10000	Total integration time (in ns).
CalculateYields	<bool>	false	Whether to output time-evolution (false) or quantum yields (true).
TransitionYields	<bool>	false	Whether to calculate quantum yields per transition (true) or per State defined in the SpinSystem (false). Quantum yield calculations only.
OutputStride	<integer>	1	How often output should be written (time-evolution output only), i.e. write output every n th numerical integration timestep, where the parameter n is specified here.

Technical details

Solves the Liouville-von Neumann equation for the density operator ρ :

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[\mathbf{H}, \rho] + \mathcal{K}(\rho), \quad (2)$$

where \mathbf{H} is the (time-dependent) Hamiltonian, and \mathcal{K} is the superoperator representing reactions (either Haberkorn or Lindblad form).

The time-evolution output is the expectation value of state projection operators:

$$p_i(t) = \text{Tr}(\mathbf{P}_i \rho(t)), \quad (3)$$

where \mathbf{P}_i is the expectation value of state i . This is calculated for each State object defined in the SpinSystem (i.e. the index i refers to such a State object).

If quantum yields are requested instead, there are two possibilities; by default the yields are just calculated for each State object defined in the SpinSystem:

$$\Phi_i = \int_0^{\text{TotalTime}} \text{Tr}(\mathbf{P}_i \rho(t)) dt. \quad (4)$$

Note that the upper integration limit is *TotalTime* rather than ∞ , and that no rate constant is involved here; thus this is not a “proper yield”. The other possibility is when the *TransitionYields* parameter is true:

$$\Phi_j = k_j \int_0^{\text{TotalTime}} \text{Tr}(\mathbf{P}_j \rho(t)) dt. \quad (5)$$

Here the index j refers to the *SourceState* parameter of a transition object (which is also a State object, and hence \mathbf{P}_j is the projection onto that state) and k_j the rate of the transition. This is calculated for each transition object in the SpinSystem.

The numerical integration of the Liouville-von Neuman equation is performed using an asynchronous leapfrog algorithm to obtain the time-evolution of the spin system, and the quantum yields are calculated using the trapezoidal rule on the time-evolution data.

5.4 Task class: PeriodicHS-TimeEvolution

Brief description

Same method as DynamicHS-TimeEvolution except that periodicity in the time-dependence of the Hamiltonian is exploited to provide a faster method.

Limitations, approximations and comments

- Use this method if you use one or more time-dependent interactions, where all of these time-dependences are periodic.
- The *totaltime* parameter should be at least a few periods long in order to get any advantage from this method.
- Requires more memory than DynamicHS-TimeEvolution.
- The method does not work with trajectories.
- Assumes the Hamiltonian to be piecewise constant for very short time intervals as defined by the *StepsPerPeriod* parameter. The accuracy of the results can generally be improved by increasing this parameter (at the cost of speed and increased memory usage).
- Make sure to use a small enough timestep to avoid significant numerical errors.
- When calculating quantum yields, make sure the *totaltime* parameter is long enough to let the SpinSystem decay completely.

Parameters

Parameter	Values	Default	Description
TimeStep	<float>	0.01	Numerical integration timestep (in ns).
TotalTime	<float>	10000	Total integration time (in ns).
CalculateYields	<bool>	false	Whether to output time-evolution (false) or quantum yields (true).
TransitionYields	<bool>	false	Whether to calculate quantum yields per transition (true) or per State defined in the SpinSystem (false). Quantum yield calculations only.
OutputStride	<integer>	1	How often output should be written (time-evolution output only), i.e. write output every n th numerical integration timestep, where the parameter n is specified here.
Steps StepsPerPeriod	<integer>	50	Splits each period into this number of intervals. The Hamiltonian is assumed to be constant in each of these intervals.

Technical details

Same method as DynamicHS-TimeEvolution except that the Hamiltonian is calculated at different times before the numerical integration procedure is started, and these Hamiltonians are then cached such that they can be used repeatedly during the numerical integration; i.e. this method gains speed at the cost of increased memory usage (calculating Hamiltonians is time consuming). The period is inferred from the interactions in the spin system.

If the StepsPerPeriod parameter is denoted N and the period of the Hamiltonian by T , then the Hamiltonian is calculated at times $nT/N \forall n \in \{0, 1, \dots, N\}$ and is assumed to be constant for the duration T/N .

5.5 Task class: MultiDynamicHS-TimeEvolution

Brief description

Multi-system version of the DynamicHS-TimeEvolution method, i.e. supports transitions between SpinSystems.

Limitations, approximations and comments

- This is the most general method to compute time-evolution; it can handle all types of input.
- Use DynamicHS-TimeEvolution instead if you only have a single SpinSystem.
- This method can be used to calculate quantum yields as well: just create an extra spin system to hold the product states and create Transition objects with these product states as target (if no interactions are present in this other system the reaction products will just accumulate and so you get the accumulated quantum yield over time).
- Make sure to use a small enough timestep to avoid significant numerical errors.
- This is a very slow method; use it only when faster methods do not suffice.
- Alternative type name: *DynamicHS-MultiSystem*.

Parameters

Parameter	Values	Default	Description
TimeStep	<float>	0.01	Numerical integration timestep (in ns).
TotalTime	<float>	10000	Total integration time (in ns).
OutputStride	<integer>	1	How often output should be written (time-evolution output only), i.e. write output every n th numerical integration timestep, where the parameter n is specified here.

Technical details

To be added...

5.6 Task class: StaticSS-TimeEvolution

Brief description

Superoperator space (Liouville space) method that calculates the time-evolution of spin systems with a time-independent Hamiltonian.

Limitations, approximations and comments

- Superoperator space methods scale much worse with system size than native Hilbert space methods (in terms of both memory and speed).
- Ignores time-dependences: If time-dependences are present, they are evaluated only at time 0.

Parameters

Parameter	Values	Default	Description
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
TotalTime	<float>	10000	Total integration time (in ns).
ReactionOperator	<i>Haberkorn</i> , <i>Lindblad</i>	<i>Haberkorn</i>	The default reaction operator type to use for this task. This is ignored for Transition objects where the reaction operator type is specified.

Technical details

Calculates the propagator in superoperator space (Liouville space):

$$\hat{U} = \exp\left(-\frac{i}{\hbar}\hat{\mathbf{H}}\Delta t + \hat{\mathbf{K}}\Delta t\right) = \exp(\hat{\mathbf{A}}\Delta t), \quad (6)$$

where $\hat{\mathbf{H}}$ is the Hamiltonian superoperator, $\hat{\mathbf{K}}$ is the superoperator representing reactions (either Haberkorn or Lindblad form), and $\hat{\mathbf{A}}$ is the total Liouvillian superoperator. The propagator is then used to calculate the time-evolution:

$$\rho(t + \Delta t) = \hat{U}\rho(t) \quad (7)$$

The time-evolution output is the expectation value of state projection operators:

$$p_i(t) = \text{Tr}(\mathbf{P}_i\rho(t)), \quad (8)$$

where \mathbf{P}_i is the expectation value of state i . This is calculated for each State object defined in the SpinSystem (i.e. the index i refers to such a State object).

5.7 Task class: PeriodicSS-TimeEvolution

Brief description

Superoperator space (Liouville space) method that calculates the time-evolution of spin systems with a periodic time-dependence of the Hamiltonian. Very similar to StaticSS-TimeEvolution.

Limitations, approximations and comments

- Use this method if you use one or more time-dependent interactions, where all of these time-dependences are periodic.
- The totaltime parameter should be at least a few periods long in order to get any advantage from this method.
- Requires more memory than StaticSS-TimeEvolution, but can handle periodic time-dependences.
- The method does not work with trajectories.

- Assumes the Hamiltonian to be piecewise constant for very short time intervals as defined by the StepsPerPeriod parameter. The accuracy of the results can generally be improved by increasing this parameter (at the cost of speed and increased memory usage).
- Superoperator space methods scale much worse with system size than native Hilbert space methods (in terms of both memory and speed).

Parameters

Parameter	Values	Default	Description
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
TotalTime	<float>	10000	Total integration time (in ns).
ReactionOperator	<i>Haberkorn</i> , <i>Lindblad</i>	<i>Haberkorn</i>	The default reaction operator type to use for this task. This is ignored for Transition objects where the reaction operator type is specified.
Steps StepsPerPeriod	<integer>	50	Splits each period into this number of intervals. The Hamiltonian is assumed to be constant in each of these intervals.

Technical details

Same propagator-based method as StaticSS-TimeEvolution, except that multiple propagators are calculated: A full period T of the periodic Hamiltonian is divided into N intervals (given by the StepsPerPeriod parameter), each of length T/N , and a propagator is calculated for each interval:

$$\hat{U}(nT/N) = \exp\left(\left[-\frac{i}{\hbar}\hat{\mathbf{H}}(nT/N) + \hat{\mathbf{K}}\right]\Delta t\right) \quad \forall n \in \{0, 1, \dots, N\}, \quad (9)$$

These propagators are then used repeatedly to propagate the state of the spin system.

Note that N propagators are calculated and stored in memory, in contrast to a single propagator for the StaticSS-TimeEvolution method. This results in a significantly poorer performance than StaticSS-TimeEvolution.

5.8 Task class: MultiStaticSS-TimeEvolution

Brief description

Superoperator space (Liouville space) method. Multi-system version of StaticSS-TimeEvolution, i.e. supports transitions between SpinSystems.

Limitations, approximations and comments

- Use a different method if you only have a single spin system.
- Both source and target state of Transition objects must be completely described (i.e. use state objects where the state of *every* spin is specified). Note that this limitation is not present in the MultiDynamicHS-TimeEvolution method.
- The ReactionOperator parameter only affects the used decay operator, not the creation operator (see technical details).

- This method can be used to calculate quantum yields as well: just create an extra spin system to hold the product states and create Transition objects with these product states as target (if no interactions are present in this other system the reaction products will just accumulate and so you get the accumulated quantum yield over time).
- Superoperator space methods scale much worse with system size than native Hilbert space methods (in terms of both memory and speed).
- Alternative type name: *StaticSS-MultiSystem*.

Parameters

Parameter	Values	Default	Description
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
TotalTime	<float>	10000	Total integration time (in ns).
ReactionOperator	<i>Haberkorn</i> , <i>Lindblad</i>	<i>Haberkorn</i>	The default reaction operator type to use for this task. This is ignored for Transition objects where the reaction operator type is specified.

Technical details

To be added...

5.9 Task class: StaticSS

Brief description

Superoperator space (Liouville space) method that calculates the quantum yield. This is the most general quantum yield calculation method to use when there are no time-dependent interactions or time-dependent rate constants, since time-dependences are not taken into account.

Limitations, approximations and comments

- Superoperator space methods scale much worse with system size than native Hilbert space methods (in terms of both memory and speed).
- Ignores time-dependences: If time-dependences are present, they are evaluated only at time 0.
- Alternative type name: *StaticIVP*.

Parameters

Parameter	Values	Default	Description
TransitionYields	<bool>	false	Whether to calculate quantum yields per transition (true) or per State defined in the SpinSystem (false).
ReactionOperator	<i>Haberkorn</i> , <i>Lindblad</i>	<i>Haberkorn</i>	The default reaction operator type to use for this task. This is ignored for Transition objects where the reaction operator type is specified.

Technical details

Solves the Liouville-von Neumann equation for the density operator ρ , in superoperator space (Liouville space):

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\hat{\mathbf{H}}\rho + \hat{\mathbf{K}}\rho = \hat{\mathbf{A}}\rho, \quad (10)$$

where $\hat{\mathbf{H}}$ is the Hamiltonian superoperator, $\hat{\mathbf{K}}$ is the superoperator representing reactions (either Haberkorn or Lindblad form), and $\hat{\mathbf{A}}$ is the total Liouvillian superoperator.

There are two possibilities for calculation of the quantum yields; by default the yields are just calculated for each State object defined in the SpinSystem:

$$\Phi_i = \int_0^\infty \text{Tr}(\mathbf{P}_i\rho(t)) dt = \text{Tr}(\mathbf{P}_i\hat{\mathbf{A}}^{-1}\rho(0)). \quad (11)$$

Note that there is no rate constant is involved here; thus this is not a “proper yield”. The other possibility is when the *TransitionYields* parameter is true:

$$\Phi_j = k_j \int_0^\infty \text{Tr}(\mathbf{P}_j\rho(t)) dt = k_j \text{Tr}(\mathbf{P}_j\hat{\mathbf{A}}^{-1}\rho(0)). \quad (12)$$

Here the index j refers to the *SourceState* parameter of a transition object (which is also a State object, and hence \mathbf{P}_j is the projection onto that state) and k_j the rate of the transition. This is calculated for each transition object in the SpinSystem.

Here it is used that:

$$\int_0^\infty \rho(t) dt = -\hat{\mathbf{A}}^{-1}\rho(0), \quad (13)$$

as can be derived using the properties of the Laplace transformation.

5.10 Task class: StaticHS-SymmetricDecay

Brief description

Efficient quantum yield method relying on some assumptions.

Limitations, approximations and comments

- The interactions should be time-independent (otherwise they are just evaluated at time 0).
- Only a single spin-independent reaction is present (i.e. “symmetric decay” as all states decay with same rate).
- Only the task class RP-SymmetricUncoupled is faster and more efficient than this method.
- Alternative type name: *StaticHS*.

Parameters

Parameter	Values	Default	Description
DecayRate	<float>	0.001	The spin-independent rate constant to use (in ns ⁻¹). If the rate constant is not specified, the rate of the first defined Transition object in each spin system is used. If no Transition objects are defined in a spin system, the default value will be used.
ReactionRate			
RateConstant			

Technical details

Calculates the quantum yield by solving the equation:

$$\Phi_i = \sum_{n,m} \langle n | \mathbf{P}_i | m \rangle \langle m | \rho(0) | n \rangle \frac{k^2}{k^2 + (\omega_n - \omega_m)^2}, \quad (14)$$

where $\rho(0)$ is the initial state density operator, ω_n is an eigenvalue of the Hamiltonian (upto a factor of \hbar), $|n\rangle$ and $|m\rangle$ eigenstates of the Hamiltonian, k is the spin-independent rate constant, and \mathbf{P}_i is the projection operator onto state i . Quantum yields Φ_i are calculated for all State objects defined in the spin system.

5.11 Task class: RP-SymmetricUncoupled

Brief description

Very efficient quantum yield method subject to rather strict limitations.

Limitations, approximations and comments

- Works only for radical pairs.
- The interactions should be time-independent (otherwise they are just evaluated at time 0).
- Only a single spin-independent reaction is present (i.e. “symmetric decay” as all states decay with same rate).
- The two radical should be uncoupled (i.e. no double-spin interaction such as the exchange or magnetic dipole-dipole interaction between the two unpaired electrons).
- Which spins belong to each radical is inferred based on the “type” parameter of the spins: only two spins should be given the type “electron” (one per radical).
- This method calculates only the singlet quantum yield for the radical pair, and the triplet yield is then inferred as these two yields should add up to 1.
- Alternative type name: *RP-Uncoupled*.

Parameters

Parameter	Values	Default	Description
Tolerance	<float>	1e-10	Quantum yield contributions smaller than this tolerance level are neglected.
DecayRate ReactionRate RateConstant	<float>	0.001	The spin-independent rate constant to use (in ns ⁻¹). If the rate constant is not specified, the rate of the first defined Transition object in each spin system is used. If no Transition objects are defined in a spin system, the default value will be used.

Technical details

The two radicals are treated independently and with their own single-radical spin Hamiltonians. The first radical is referred to as radical A , and the other radical as B . The method calculates the quantum yield by solving the equation:

$$\Phi_i = \frac{1}{4} + N \sum_{p,q} \sum_{i \neq j} \sum_{g \neq h} R_{pq}^A R_{pq}^B \frac{k^2}{k^2 + (\omega_i^A - \omega_j^A + \omega_g^B - \omega_h^B)^2}, \quad (15)$$

where ω_i^m is an eigenvalue of the single-radical Hamiltonian for radical m (upto a factor of \hbar), k is the spin-independent rate constant, and R_{pq}^m is defined by:

$$R_{pq}^m = \sum_{i,j} \text{Tr} (\mathbf{S}_{m,p} |i\rangle \langle i| \mathbf{S}_{m,q} |j\rangle \langle j|), \quad (16)$$

where $|i\rangle$ are the eigenstates of the spin Hamiltonian for radical m , and $\mathbf{S}_{m,p}$ are the spin operators for the unpaired electronic spin of radical m with $p \in \{x, y, z\}$.

5.12 Task class: Gamma-Compute

Brief description

Implementation of the γ -COMPUTE algorithm. Similar to PeriodicHS-TimeEvolution except that an average of the phase of the periodic Hamiltonian is also performed, and this method only calculates quantum yields.

Limitations, approximations and comments

- *Work-in-progress...*

Parameters

Parameter	Values	Default	Description
Steps	<integer>	100	Splits each period into this number of intervals. The Hamiltonian is assumed to be constant in each of these intervals..
TotalTime Period	<float>	10000	Total integration time (in ns).

Technical details

To be added...

5.13 Task class: TaskStaticSS-Redfield

Brief description

Superoperator space (Liouville space) method that calculates the quantum yield including the Redfield relaxation operator. The construction of the relaxation operator can be executed for each interaction separately. Different options are available such as choosing alternative operator basis or constructing spectral densities in several ways.

Limitations, approximations and comments

- Superoperator space methods scale much worse with system size than native Hilbert space methods (in terms of both memory and speed).
- Calculating the relaxation matrix fastly becomes enourmously recourse demanding due to the exponential increase with system size.
- Keep in mind that Bloch-Wangsness-Redfield theory is a perturbative method which assume a Markovian process. Every calculation should be investigated carefully from a physical perspective.

Parameters

Parameter	Values	Default	Description
TransitionYields	<bool>	false	Whether to calculate quantum yields per transition (true) or per State defined in the SpinSystem (false).
ReactionOperator	<i>Haberkorn</i> , <i>Lindblad</i>	<i>Haberkorn</i>	The default reaction operator type to use for this task. This is ignored for Transition objects where the reaction operator type is specified.

Parameters in Interaction object

Technical details

General: A comprehensive derivation of BRW theory can be found in textbooks and publications [2, 3, 4]. A brief description of the essential formalism is presented here, emphasizing the numerical implementation in *MolSpin*. BRW theory describes the interaction between a spin system S and the environment or bath B. Mathematically speaking, the total Hamiltonian \hat{H} of the spin system and the bath can be factorized as [3]:

$$\hat{H} = \hat{H}_S + \hat{H}_B + \hat{H}_I(t). \quad (17)$$

Here, \hat{H}_S is the static spin system Hamiltonian, \hat{H}_B is the bath Hamiltonian, and \hat{H}_I describes the time-dependent interaction between the bath and the spins. The goal of BRW theory is to neglect the explicit description of the bath dynamics, \hat{H}_B , focus on the spin dynamics of the system and modify \hat{H}_I in such a way that all time-dependence can eventually be described by a simple coupling function.

Assuming that the ensemble-averaged expectation value of \hat{H}_I is zero, that the bath is unaffected by the system dynamics, and that the time-dependent perturbation has a significantly lower amplitude than the static Hamiltonian \hat{H}_S , the evolution of the system can be formulated in the interaction picture, up to second-order in \hat{H}_I , as [3]:

$$\frac{d\hat{\rho}_S(t)}{dt} = - \int_0^t dt' \text{Tr}_B[\hat{H}_I(t), [\hat{H}_I^\dagger(t'), \hat{\rho}_S(t') \otimes \hat{\rho}_B]], \quad (18)$$

where $\hat{\rho}_S(t)$ is the density operator of the spin system, $\hat{\rho}_B$ is the density operator of the bath, Tr_B is the trace over the bath states. In the following all derivations are in the interaction picture unless otherwise stated. Equation (18) rests on the assumption (often called the Born approximation) that the complete density operator $\hat{\rho}(t)$ (system and bath) can be factorized as [3]:

$$\hat{\rho}(t) \approx \hat{\rho}_S(t) \otimes \hat{\rho}_B. \quad (19)$$

Equation (18) is non-Markovian insofar as the change in the density matrix at a time t depends on the previous time instances $t' < t$. A further simplification of Eq. (18) postulates a Markovian behavior of the spin system (Markov approximation). This is realized, by first assuming that $\hat{\rho}_S(t)$ changes slowly with t , so that Eq. (18) can be rewritten [3]:

$$\frac{d\hat{\rho}_S(t)}{dt} = - \int_0^t dt' \text{Tr}_B[\hat{H}_I(t), [\hat{H}_I^\dagger(t'), \hat{\rho}_S(t) \otimes \hat{\rho}_B]]. \quad (20)$$

Although this equation is local in time, it is still not Markovian due to the time evolution of the density matrix $\hat{\rho}_S(t)$, which depends on the explicit choice of the initial state $\hat{\rho}_S(0)$ at $t = 0$. Equation (20) can be formulated in Markovian form if the substitution $t' \rightarrow t - t'$ is made and the upper limit of the integral is extended to ∞ :

$$\frac{d\hat{\rho}_S(t)}{dt} = - \int_0^\infty dt' \text{Tr}_B[\hat{H}_I(t), [\hat{H}_I^\dagger(t - t'), \hat{\rho}_S(t) \otimes \hat{\rho}_B]]. \quad (21)$$

Table 1: Parameters available for *interaction-object* when using BRW-theory.

Parameter	Values	Default	Description
tau_c	double vector	/ -	Defining correlation times for a relaxation process. It is possible to use one value or a list of values separated by commas.
g	double vector	/ -	Defining amplitudes of correlation functions for relaxation processes. It is possible to use one value or a list of values separated by commas.
def_multexpo	0/1	0	(0) The general approach is used for the syntax of tau_c and g (1) is used when several multi-exponential fits must be considered for all operator pairs. I.e. for a double spin interaction (with Cartesian operators \hat{S}_1n, \hat{S}_2m with 9 coupling elements a total of 81 correlation functions is required to map the complete relaxation effect in the interaction. The input syntax of tau_c and g changes to a list of lists in the following way: $g = [g_{xx,xx1}, g_{xx,xx2}, \dots], [g_{xx,xy1}, g_{xx,xy2}, \dots], [g_{xx,xz1}, g_{xx,xz2}, \dots], \dots$. Each [...] resembles a multi-exponential fit (amplitude g/tau_c) of a certain coupling element with another coupling element.
def_g	0/1	0	(0) one general amplitude/amplitude-vector is chosen for the construction of spectral densities. <i>Note:</i> When choosing a list of amplitudes an equal number of correlation times <i>tau_c</i> must be used. (1) An amplitude value is set for each operator specifically. Spectral density is constructed with correlation between two operators by using the product of operator amplitudes.
def_specdens	0/1	0	(0) General Lorentzian spectral densities are calculated including the imaginary parts. (1) Neglecting the imaginary parts (see <i>Technical details</i>).
ops	0/1	0	(0) Irreducible spherical tensors of rank 0 and rank 2 are used to define the interactions. (1) Cartesian operators are used to define the interactions.
terms	0/1	0	(0) All cross terms between operators (ops) are calculated. (1) Cross terms between operators (ops) are neglected.
coeff	0/1	0	Use only with irreducible spherical tensors: (0) No modulation of amplitude with interaction coupling tensor values is used. (1) Spatial tensors are constructed for each irreducible spherical tensor using interaction coupling tensor.
slip	0/1	0	Modify initial condition (density matrix) with slippage operator according to P. Gaspard and M. Nagaoka. [1]

In general, the interaction operator \hat{H}_I can be decomposed into a sum of direct products of operators \hat{S}_α describing the spin system and operators \hat{B}_α describing the bath:

$$\hat{H}_I(t) = \sum_{\alpha} \hat{S}_{\alpha}(t) \otimes \hat{B}_{\alpha}(t), \quad (22)$$

in which \hat{S}_α and \hat{B}_α can be chosen arbitrarily as long as $\hat{H}_I(t) = \hat{H}_I^\dagger(t)$. Equation (21) can now be rewritten by formulating the commutators in their explicit form with $\hat{H}_I(t-t') = \hat{H}_I^\dagger(t-t')$:

$$\begin{aligned} \frac{d\hat{\rho}_S(t)}{dt} &= -\sum_{\alpha,\beta} \int_0^\infty dt' g_{\alpha\beta}(t') \left(\hat{S}_\alpha(t) \hat{S}_\beta^\dagger(t-t') \hat{\rho}_S(t) - \hat{S}_\beta^\dagger(t-t') \hat{\rho}_S(t) \hat{S}_\alpha(t) \right) \\ &\quad + g_{\alpha\beta}^*(t') \left(\hat{\rho}_S(t) \hat{S}_\beta(t-t') \hat{S}_\alpha^\dagger(t) - \hat{S}_\alpha^\dagger(t) \hat{\rho}_S(t) \hat{S}_\beta(t-t') \right), \end{aligned} \quad (23)$$

where $g_{\alpha\beta}(t') = \text{Tr}_B(\hat{B}_\alpha(t) \hat{B}_\beta^\dagger(t-t') \hat{\rho}_B) = \langle \hat{B}_\alpha(t) \hat{B}_\beta^\dagger(t-t') \rangle$ is the bath correlation function. If the $g_{\alpha\beta}(t')$ are homogeneous in time:

$$\langle \hat{B}_\alpha(t) \hat{B}_\beta^\dagger(t-t') \rangle = \langle \hat{B}_\alpha(t') \hat{B}_\beta^\dagger(0) \rangle. \quad (24)$$

Equation (23) can now be formulated in matrix form in the eigenbasis of \hat{H}_S :

$$S_{mn}^\alpha(t) = \langle n | \hat{S}_\alpha(t) | m \rangle = S_{mn}^\alpha e^{i\omega_{mn}t}, \quad (25)$$

where $\omega_{mn} = \epsilon_m - \epsilon_n$ and ϵ_m is the eigenvalue of \hat{H}_S corresponding to the eigenstate $|m\rangle$. Equation (23) can now be rewritten in the Schrödinger picture as [3] (the subscript S is omitted here):

$$\begin{aligned} \frac{d\rho_{ab}(t)}{dt} &= -i\omega_{ab}\rho_{ab}(t) - \sum_{\alpha,\beta} \sum_{c,d} \int_0^\infty dt' \{ g_{\alpha\beta}(t') [\delta_{b,d} \sum_n S_{an}^\alpha S_{cn}^{\beta*} e^{i\omega_{cn}t'} - S_{ca}^{\beta*} S_{db}^\alpha e^{i\omega_{ca}t'}] \\ &\quad + g_{\alpha\beta}^*(t') [\delta_{a,c} \sum_n S_{dn}^\beta S_{bn}^{\alpha*} e^{i\omega_{dn}t'} - S_{ca}^{\alpha*} S_{db}^\beta e^{i\omega_{bd}t'}] \} \rho_{cd}(t). \end{aligned} \quad (26)$$

The first term on the right hand arises from the static Hamiltonian \hat{H}_S while the integral accounts for the spin relaxation induced by the system-bath interactions. Equation (26) can be rewritten in terms of the spectral densities $J_{\alpha\beta}(\omega_{mn})$ [3]:

$$J_{\alpha\beta}(\omega_{mn}) = \int_0^\infty dt' g_{\alpha\beta}(t') e^{i\omega_{mn}t'} \quad (27)$$

which represent the strength of the system-bath coupling at frequency ω_{mn} . Equation (26) thus assumes the form [5]:

$$\frac{d\rho_{ab}}{dt} = -i\omega_{ab}\rho_{ab} + \sum_{c,d} R_{abcd}\rho_{cd}, \quad (28)$$

where R_{abcd} is given by:

$$\begin{aligned} R_{abcd} &= -\sum_{\alpha,\beta} [\delta_{b,d} \sum_n S_{an}^\alpha S_{cn}^{\beta*} J_{\alpha\beta}(\omega_{cn}) - S_{ca}^{\beta*} S_{db}^\alpha J_{\alpha\beta}(\omega_{ca}) \\ &\quad + \delta_{a,c} \sum_n S_{dn}^\beta S_{bn}^{\alpha*} J_{\alpha\beta}^*(\omega_{dn}) - S_{ca}^{\alpha*} S_{db}^\beta J_{\alpha\beta}^*(\omega_{db})]. \end{aligned} \quad (29)$$

Including the reaction operator and transforming Eq. (28) into Liouville space, one obtains the following equation of motion for the spin density operator:

$$\frac{d\hat{\rho}(t)}{dt} = -i\hat{H}_S\hat{\rho}(t) + \hat{\mathcal{K}}\hat{\rho}(t) + \hat{R}\hat{\rho}(t) = \hat{L}\hat{\rho}(t), \quad (30)$$

where \hat{L} is the total Liouvillian superoperator. *MolSpin* solves Eq. (30) numerically. There are two possibilities for the calculation of quantum yields Φ_i ; by default the yields are just calculated for each State object defined in the SpinSystem:

$$\Phi_i = \int_0^\infty \text{Tr}(\mathbf{P}_i \rho(t)) dt = \text{Tr}(\mathbf{P}_i \hat{\mathbf{L}}^{-1} \rho(0)). \quad (31)$$

Note that there is no rate constant is involved here; thus this is not a “proper yield”. The other possibility is when the *TransitionYields* parameter is true:

$$\Phi_j = k_j \int_0^\infty \text{Tr}(\mathbf{P}_j \rho(t)) dt = k_j \text{Tr}(\mathbf{P}_j \hat{\mathbf{L}}^{-1} \rho(0)). \quad (32)$$

Here the index j refers to the *SourceState* parameter of a transition object (which is also a State object, and hence \mathbf{P}_j is the projection onto that state) and k_j the rate of the transition. This is calculated for each transition object in the SpinSystem. Here it is used that:

$$\int_0^\infty \rho(t) dt = -\hat{\mathbf{L}}^{-1} \rho(0), \quad (33)$$

as can be derived using the properties of the Laplace transformation.

Spectral densities: For the solution of the spectral densitie a fast decaying correlation function in the following form is assumed:

$$g(t') = g(0) e^{-\frac{t'}{\tau_c}}. \quad (34)$$

Here τ_c is the correlation time and $g(0)$ is the amplitude of the correlation. By integrating Eq. (34) according to Eq. (27), the spectral density $J(\omega)$ then reads:

$$J(\omega) = g(0) \frac{1}{\frac{1}{\tau_c} - i\omega} = g(0) \left(\frac{\tau_c}{1 + \omega^2 \tau_c^2} + \frac{i\omega \tau_c^2}{1 + \omega^2 \tau_c^2} \right). \quad (35)$$

The imaginary term on the right-hand side of the equation is the dynamic frequency shift, which is often neglected when studying relaxation phenomena due to its small contribution. *MolSpin* has the option to neglect this term.

In many realistic situations, such as the motion of atoms in proteins, the spectral density cannot accurately be described by a single exponential correlation function [6]. A more realistic description is achieved by constructing the spectral density from a linear combination of correlation functions [6]:

$$J(\omega) = \sum_q g_q(0) \frac{1}{\frac{1}{\tau_c^q} - i\omega}, \quad (36)$$

where $g_q(0)$ and τ_c^q are the amplitudes and correlation times, respectively [6]. The user can specify a list of amplitudes $g(0)$ and correlation times τ_c received from the fit of a correlation function in the parameters g and tau_c with a "," as delimiter:

```

1 Interaction HF
2 {
3     type = Hyperfine;
4     group1 = electron1;
5     group2 = nucleus1;
6     tensor = isotropic("43.99"); //Coupling in Tesla [T]
7     tau_c = 0.0098, 0.0066, 0.0040, 0.03; // Correlation times in nanoseconds [ns]
8     g = 0.03, 0.02, 0.01, 0.94; // Correlation function list
9 }

```

Note, that an equal amount of $g(0)$ and τ_c is required.

Operator basis: Depending on the problem of interest different spin operators S_α are most suitable as a basis for the construction of R_{abcd} in Eq. (29). For instance, when relaxation results from rotational motion, a spherical tensor basis \hat{T} for the interaction of interest is recommended, while for more general problems the usual Cartesian spin operators \hat{S}_x, \hat{S}_y and \hat{S}_z may be used [5, 4]. *MolSpin*

provides both basis sets. The spherical tensor formalism relies on the decomposition of linear and bilinear interactions into spatial tensors a_q^k and spherical tensors \hat{T}_q^k of rank k with $q = -k, \dots, 0, \dots, k$ [4]:

$$\hat{L} \cdot \mathbf{A} \cdot \hat{S} = \sum_k^{k_{max}} \sum_{q=-k}^{+k} (-1)^q a_{-q}^k \hat{T}_q^k. \quad (37)$$

Here, \hat{L} and \hat{S} are the vectors of Cartesian spin operators for bilinear interactions, and \mathbf{A} is the coupling matrix. In the case of linear interactions \hat{L} is replaced by a field vector \vec{F} . The terms of rank $k = 1$ are usually zero due to \mathbf{A} being symmetric [4]. In *MolSpin*, the spatial and spherical tensors are as follows:

$$\hat{T}_0^0 = \frac{1}{\sqrt{3}}[\hat{T}_{xixj} + \hat{T}_{yiyj} + \hat{T}_{zizj}], \quad a_0^0 = \frac{1}{\sqrt{3}}[a_{xx} + a_{yy} + a_{zz}], \quad (38)$$

$$\hat{T}_0^2 = \frac{1}{\sqrt{6}}[3\hat{T}_{zizj} - (\hat{T}_{xixj} + \hat{T}_{yiyj} + \hat{T}_{zizj})], \quad a_0^2 = \frac{1}{\sqrt{6}}[3a_{zz} - (a_{xx} + a_{yy} + a_{zz})], \quad (39)$$

$$\hat{T}_{\pm 1}^2 = \mp \frac{1}{2}[\hat{T}_{xizj} + \hat{T}_{zixj} \pm i(\hat{T}_{yizj} + \hat{T}_{ziyj})], \quad a_{\pm 1}^2 = \mp \frac{1}{2}[a_{xz} + a_{zx} \pm i(a_{yz} + a_{zy})], \quad (40)$$

$$\hat{T}_{\pm 2}^2 = \frac{1}{2}[\hat{T}_{xixj} - \hat{T}_{yiyj} \pm i(\hat{T}_{xijy} + \hat{T}_{yijx})], \quad a_{\pm 2}^2 = \frac{1}{2}[a_{xx} - a_{yy} \pm i(a_{xy} + a_{yx})], \quad (41)$$

where \hat{T}_{ajib} is the product of spin operators \hat{S}_{ai} and \hat{L}_{bj} of the spins i and j for bilinear interactions and the product of \hat{S}_{ai} and \vec{F}_{bj} for linear interactions.

By using the parameter `def_g = 1` the user can provide each spatial tensor with a specific factor. In terms of ordering the factors in the right manner, *MolSpin* provides a fixed order of spherical tensors:

$$\hat{T}_0^0, \hat{T}_0^2, \hat{T}_{-1}^2, \hat{T}_1^2, \hat{T}_{-2}^2, \hat{T}_2^2. \quad (42)$$

Therefore, the factors can be set with $g = a_{\hat{T}_0^0}, a_{\hat{T}_0^2}, a_{\hat{T}_{-1}^2}, a_{\hat{T}_1^2}, a_{\hat{T}_{-2}^2}, g(0)_{\hat{T}_2^2}$; when using the parameter `def_g = 1`. Note, that the amplitudes of each spectral density between to tensors is constructed by the product of both factors. Additionally, when `def_g = 0` all operators will be given the same amplitude. However, in this option the amplitude is directly received for the spectral density ($g(0)$) and factors have to be squared in order to get the same results when using `def_g = 1`. When `coeff` is set to 1 spatial modulation according to equation 38-41 will be performed for each operator and will be included additionally to the selected amplitude.

Multi-exponential fits: When using the keyword `def_multexpo`, *MolSpin* requires multiple list of correlation function fits as described in Eq. 36. Depending on the operator basis and interaction (SingleSpin, DoubleSpin) chosen, up to 81 correlation function fits are required. I. e. for a DoubleSpin interaction (with Cartesian operators $\hat{S}_{1n}, \hat{S}_{2m}$ with 9 coupling elements a total of 81 correlation functions is required to map the complete relaxation effect in the interaction. The input syntax of `tau_c` and `g` changes to a list of lists in the following way:

```

1 Interaction HF
2 {
3   type = Hyperfine;
4   group1 = electron1;
5   group2 = nucleus1;
6   tau_c = ["t_xx,xx1, t_xx,xx2, ..."], ["t_xx,xy1, t_xx,xy2, ..."],
7   ["t_xx,xz1, t_xx,xz2, ..."], ...;
8
9   g = ["g_xx,xx1, g_xx,xx2, ..."], ["g_xx,xy1, g_xx,xy2, ..."],
10  ["g_xx,xz1, g_xx,xz2, ..."], ...;
11 }

```


Each ["..."] resembles a multi-exponential fit (amplitude g/τ_c) of a certain coupling element with another coupling element. The sequence of multi-exponential fits for "g =" and "tau_c =" are set in the order of couplings elements row-by-row as follows (in this example in total 81 correlation function fits):

```
["..."]xx,xx, ["..."]xx,xy, ["..."]xx,xz, ["..."]xx,yx, ["..."]xx,yy, ["..."]xx,yz, ["..."]xx,zx, ["..."]xx,zy, ["..."]xx,zz,
["..."]xy,xx, ["..."]xy,xy, ..., ["..."]xz,xx, ..., ["..."]yx,xx, ...;
```

Performance: The Redfield class is highly parallelized due to the inclusion of Armadillo [7] which guarantees fast calculations of linear algebra problems. Additionally, constructions of several *pragma* directives using OpenMP [8] are employed. Note, that the performance and thread dependency will vary with the choice of basis operators. When dealing with big spin systems the relaxation matrix becomes enormously resource demanding due to the exponential growth with respect to the considered Hilbert space. However, most of the matrix elements are zero. By employing the sparse matrix formalism integrated in the Armadillo library even bigger systems can be solved. To use the sparse matrix formalism the user just has to add *-sparse* in the *type* object:

```

1 Run
2 {
3     Task Method1
4     {
5         type = "redfield-relaxation-sparse";
6         logfile = "redfield-relaxation.log";
7         datafile = "redfield-relaxation.dat";
8         transitionyields = true;
9     }
10 }
```

However, the performance with respect to multithreading drastically decreases when using sparse matrix formalism due to the transformation of sparse matrices into normal matrices when dealing i.e. with multiplication. Thus, the user may sacrifice performance in order to investigate bigger spin systems.

5.14 Task class: TaskStaticSS-Redfield-TimeEvolution

Brief description

Superoperator space (Liouville space) method that calculates the quantum yield including the Redfield relaxation operator. The construction of the relaxation operator can be executed for each interaction separately. Different options are available such as choosing alternative operator basis or constructing spectral densities in several ways.

Limitations, approximations and comments

- Superoperator space methods scale much worse with system size than native Hilbert space methods (in terms of both memory and speed).
- Calculating the relaxation matrix fastly becomes enormously resource demanding due to the exponential increase with system size.
- Keep in mind that Redfield theory is a perturbative method which assume a Markovian process. Every calculation should be investigated carefully from a physical perspective.

Parameters

Parameter	Values	Default	Description
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
TotalTime	<float>	10000	Total integration time (in ns).
ReactionOperator	<i>Haberkorn</i> , <i>Lindblad</i>	<i>Haberkorn</i>	The default reaction operator type to use for this task. This is ignored for Transition objects where the reaction operator type is specified.

Parameters in Interaction object

Table 2: Parameters available for *interaction-object* when using BRW-theory.

Parameter	Values	Default	Description
tau_c	<i>double</i> vector	/ -	Defining correlation times for a relaxation process. It is possible to use one value or a list of values separated by commas.
g	<i>double</i> vector	/ -	Defining amplitudes of correlation functions for relaxation processes. It is possible to use one value or a list of values separated by commas.
def_multexpo	0/1	0	(0) The general approach is used for the syntax of tau_c and g (1) is used when several multi-exponential fits must be considered for all operator pairs. I.e. for a double spin interaction (with Cartesian operators \hat{S}_1n, \hat{S}_2m with 9 coupling elements a total of 81 correlation functions is required to map the complete relaxation effect in the interaction. The input syntax of tau_c and g changes to a list of lists in the following way: g = ["g _{xx,xx1} , g _{xx,xx2} , ..."], ["g _{xx,xy1} , g _{xx,xy2} , ..."], ["g _{xx,xz1} , g _{xx,xz2} , ..."], ...;. Each ["..."] resembles a multi-exponential fit (amplitude g/tau_c) of a certain coupling element with another coupling element.
def_g	0/1	0	(0) one general amplitude/amplitude-vector is chosen for the construction of spectral densities. <i>Note:</i> When choosing a list of amplitudes an equal number of correlation times <i>tau_c</i> must be used. (1) An amplitude value is set for each operator specifically. Spectral density is constructed with correlation between two operators by using the product of operator amplitudes.
def_specdens	0/1	0	(0) General Lorentzian spectral densities are calculated including the imaginary parts. (1) Neglecting the imaginary parts (see <i>Technical details</i>).
ops	0/1	0	(0) Irreducible spherical tensors of rank 0 and rank 2 are used to define the interactions. (1) Cartesian operators are used to define the interactions.
terms	0/1	0	(0) All cross terms between operators (ops) are calculated. (1) Cross terms between operators (ops) are neglected.
coeff	0/1	0	Use only with irreducible spherical tensors: (0) No modulation of amplitude with interaction coupling tensor values is used. (1) Spatial tensors are constructed for each irreducible spherical tensor using interaction coupling tensor.
slip	0/1	0	Modify initial condition (density matrix) with slippage operator according to P. Gaspard and M. Nagaoka. [1]

Technical details

For construction of the relaxation matrix we refer the reader to section 5.13. This method calculates the propagator in superoperator space (Liouville space):

$$\hat{U} = \exp\left(-\frac{i}{\hbar} \hat{\mathbf{H}} \Delta t + \hat{\mathbf{K}} \Delta t + \mathbf{R} \Delta t\right) = \exp(\hat{\mathbf{A}} \Delta t), \quad (43)$$

where $\hat{\mathbf{H}}$ is the Hamiltonian superoperator, $\hat{\mathbf{K}}$ is the superoperator representing reactions (either Haberkorn or Lindblad form), $\hat{\mathbf{R}}$ is the Redfield relaxation matrix, and $\hat{\mathbf{A}}$ is the total Liouvillian superoperator. The propagator is then used to calculate the time-evolution:

$$\rho(t + \Delta t) = \hat{\mathbf{U}}\rho(t) \quad (44)$$

The time-evolution output is the expectation value of state projection operators:

$$p_i(t) = \text{Tr}(\mathbf{P}_i\rho(t)), \quad (45)$$

where \mathbf{P}_i is the expectation value of state i . This is calculated for each State object defined in the SpinSystem (i.e. the index i refers to such a State object).

5.15 Task class: MultiStaticSS-Redfield-TimeEvolution

Brief description

Superoperator space (Liouville space) method that calculates the quantum yield including the Redfield relaxation operator for multiple spin systems. The construction of the relaxation operator can be executed for each interaction separately. Different options are available such as choosing alternative operator basis or constructing spectral densities in several ways.

Limitations, approximations and comments

- Use a different method if you only have a single spin system.
- Both source and target state of Transition objects must be completely described (i.e. use state objects where the state of *every* spin is specified). Note that this limitation is not present in the MultiDynamicHS-TimeEvolution method.
- The ReactionOperator parameter only affects the used decay operator, not the creation operator (see technical details).
- This method can be used to calculate quantum yields as well: just create an extra spin system to hold the product states and create Transition objects with these product states as target (if no interactions are present in this other system the reaction products will just accumulate and so you get the accumulated quantum yield over time).
- Superoperator space methods scale much worse with system size than native Hilbert space methods (in terms of both memory and speed).
- Calculating the relaxation matrix fastly becomes enourmously recourse demanding due to the exponential increase with system size.
- Keep in mind that Redfield theory is a perturbative method which assume a Markovian process. Every calculation should be investigated carefully from a physical perspective.

Parameters

Parameter	Values	Default	Description
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
TotalTime	<float>	10000	Total integration time (in ns).
ReactionOperator	<i>Haberkorn</i> , <i>Lindblad</i>	<i>Haberkorn</i>	The default reaction operator type to use for this task. This is ignored for Transition objects where the reaction operator type is specified.

Parameters in Interaction object

Table 3: Parameters available for *interaction-object* when using BRW-theory.

Parameter	Values	Default	Description
tau_c	<i>double</i> vector	/ -	Defining correlation times for a relaxation process. It is possible to use one value or a list of values separated by commas.
g	<i>double</i> vector	/ -	Defining amplitudes of correlation functions for relaxation processes. It is possible to use one value or a list of values separated by commas.
def_multexpo	0/1	0	(0) The general approach is used for the syntax of tau_c and g (1) is used when several multi-exponential fits must be considered for all operator pairs. I.e. for a double spin interaction (with Cartesian operators \hat{S}_1n, \hat{S}_2m with 9 coupling elements a total of 81 correlation functions is required to map the complete relaxation effect in the interaction. The input syntax of tau_c and g changes to a list of lists in the following way: $g = ["g_{xx,xx1}, g_{xx,xx2}, \dots"], ["g_{xx,xy1}, g_{xx,xy2}, \dots"], ["g_{xx,xz1}, g_{xx,xz2}, \dots"], \dots$. Each ["..."] resembles a multi-exponential fit (amplitude g/tau_c) of a certain coupling element with another coupling element.
def_g	0/1	0	(0) one general amplitude/amplitude-vector is chosen for the construction of spectral densities. <i>Note:</i> When choosing a list of amplitudes an equal number of correlation times <i>tau_c</i> must be used. (1) An amplitude value is set for each operator specifically. Spectral density is constructed with correlation between two operators by using the product of operator amplitudes.
def_specdens	0/1	0	(0) General Lorentzian spectral densities are calculated including the imaginary parts. (1) Neglecting the imaginary parts (see <i>Technical details</i>).
ops	0/1	0	(0) Irreducible spherical tensors of rank 0 and rank 2 are used to define the interactions. (1) Cartesian operators are used to define the interactions.
terms	0/1	0	(0) All cross terms between operators (ops) are calculated. (1) Cross terms between operators (ops) are neglected.
coeff	0/1	0	Use only with irreducible spherical tensors: (0) No modulation of amplitude with interaction coupling tensor values is used. (1) Spatial tensors are constructed for each irreducible spherical tensor using interaction coupling tensor.
slip	0/1	0	Modify initial condition (density matrix) with slippage operator according to P. Gaspard and M. Nagaoka. [1]

Technical details

To be added...

5.16 Task class: RP-SymmetricUncoupled-Redfield

Brief description

This method is similar to the static RP-SymmetricUncoupled method with the additional inclusion of Bloch-Redfield-Wangsness Theory. The method should be used if there are no interactions between the electrons of the radical pair to reduce the size of Hilbert space by dividing it into to sub-parts. However in contrast to RP-SymmetricUncoupled a numerical evaluation of the integral must be use in order to produce quantum yields. This make this method very slow and should only be used when many spins are included in the system and other approach cannot be used anymore.

Limitations, approximations and comments

- This method uses numerical integration to produce quantum yields. Keep in mind that this is very inefficient.
- Superoperator space methods scale much worse with system size than native Hilbert space methods (in terms of both memory and speed).
- Calculating the relaxation matrix fastly becomes enourmously recourse demanding due to the exponential increase with system size.
- Keep in mind that Redfield theory is a perturbative method which assume a Markovian process. Every calculation should be investigated carefully from a physical perspective.

Parameters

Parameter	Values	Default	Description
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
TotalTime	<float>	10000	Total integration time (in ns).
ReactionOperator	<i>Haberkorn</i> , <i>Lindblad</i>	<i>Haberkorn</i>	The default reaction operator type to use for this task. This is ignored for Transition objects where the reaction operator type is specified.

Parameters in Interaction object

Table 4: Parameters available for *interaction-object* when using BRW-theory.

Parameter	Values	Default	Description
tau_c	double vector	/ -	Defining correlation times for a relaxation process. It is possible to use one value or a list of values separated by commas.
g	double vector	/ -	Defining amplitudes of correlation functions for relaxation processes. It is possible to use one value or a list of values separated by commas.
def_multexpo	0/1	0	(0) The general approach is used for the syntax of tau_c and g (1) is used when several multi-exponential fits must be considered for all operator pairs. I.e. for a double spin interaction (with Cartesian operators \hat{S}_1n, \hat{S}_2m with 9 coupling elements a total of 81 correlation functions is required to map the complete relaxation effect in the interaction. The input syntax of tau_c and g changes to a list of lists in the following way: g = ["g _{xx,xx1} , g _{xx,xx2} , ..."], ["g _{xx,xy1} , g _{xx,xy2} , ..."], ["g _{xx,xz1} , g _{xx,xz2} , ..."], ...;. Each ["..."] resembles a multi-exponential fit (amplitude g/tau_c) of a certain coupling element with another coupling element.
def_g	0/1	0	(0) one general amplitude/amplitude-vector is chosen for the construction of spectral densities. <i>Note:</i> When choosing a list of amplitudes an equal number of correlation times <i>tau_c</i> must be used. (1) An amplitude value is set for each operator specifically. Spectral density is constructed with correlation between two operators by using the product of operator amplitudes.
def_specdens	0/1	0	(0) General Lorentzian spectral densities are calculated including the imaginary parts. (1) Neglecting the imaginary parts (see <i>Technical details</i>).
ops	0/1	0	(0) Irreducible spherical tensors of rank 0 and rank 2 are used to define the interactions. (1) Cartesian operators are used to define the interactions.
terms	0/1	0	(0) All cross terms between operators (ops) are calculated. (1) Cross terms between operators (ops) are neglected.
coeff	0/1	0	Use only with irreducible spherical tensors: (0) No modulation of amplitude with interaction coupling tensor values is used. (1) Spatial tensors are constructed for each irreducible spherical tensor using interaction coupling tensor.
slip	0/1	0	Modify initial condition (density matrix) with slippage operator according to P. Gaspard and M. Nagaoka. [1]

Technical details

To be added...

5.17 Task class: HamiltonianEigenvalues

Brief description

Calculates eigenvalues of the spin Hamiltonian. Can also calculate resonance frequencies and other properties of the Hamiltonian (see parameters).

Limitations, approximations and comments

- The resonance frequency analysis is a secondary result and is therefore printed in the logfile rather than the datafile.
- Alternative type name: *Eigenvalues*.

Parameters

Parameter	Values	Default	Description
Superspace UseSuperspace	<bool>	false	Whether to use the superspace Hamiltonian rather than the native Hilbert space (“normal”) Hamiltonian for all calculations.
Eigenvectors PrintEigenvectors	<bool>	false	Whether eigenvectors should be printed.
Hamiltonian PrintHamiltonian	<bool>	false	Whether the Hamiltonian should be printed.
SeparateReal SeparateImaginary SeparateComplex	<bool>	false	Whether output should be written as complex numbers (false) or as separated real and imaginary parts (true).
ResonanceFrequencies Frequencies Resonances	<bool>	false	Whether to perform a resonance frequency calculation.
SpinList	<Spin list>	<i>empty</i>	A comma-separated list of spins. Transition matrix elements will be calculated for these spins if a resonance frequency analysis is performed.
InitialTime StartTime Begin	<float>	0	The initial time (in ns) used for all calculations.
TotalTime StopTime End	<float>	0	If this is larger than the initial time, all calculations will be performed at times from initial to total time (in ns), as defined by the timestep parameter.
Timestep	<float>	1	Timestep (in ns) to use if a totaltime larger than initialtime is specified.
ReferenceStates RefStates	<State list>	<i>empty</i>	Comma-separated list of states. The projection of each eigenstate onto these states is calculated.

Technical details

The eigenvalues and eigenstates are calculated by diagonalization of the Hamiltonian at a given time (time 0 by default, but the calculations can be done for different times using the initialtime, totaltime and timestep parameters).

Resonance frequencies are calculated as the difference between pairs of eigenvalues, and are calculated for each pair of eigenvalues. If a spinlist is specified, the following transition matrix elements are also calculated for each specified spin:

$$T_{i,x} = \langle v_p | \mathbf{S}_{i,x} | v_q \rangle, \quad T_{i,y} = \langle v_p | \mathbf{S}_{i,y} | v_q \rangle, \quad T_{i,z} = \langle v_p | \mathbf{S}_{i,z} | v_q \rangle, \quad (46)$$

where i is the spin index from the spinlist, $|v_p\rangle$ and $|v_q\rangle$ are eigenstates of the Hamiltonian, $\mathbf{S}_{i,r}$, $r \in \{x, y, z\}$ are the spin operators, and $T_{i,r}$, $r \in \{x, y, z\}$ are the transition matrix elements.

The reference states, if specified, provide the projections of eigenstate onto these references, i.e.:

$$R_{i,p} = \langle v_p | \mathbf{P}_i | v_p \rangle, \quad (47)$$

where $R_{i,p}$ is the projection of the p th eigenstate onto reference state i , with corresponding state projection operator \mathbf{P}_i and eigenstate $|v_p\rangle$. This is calculated for all eigenstates.

5.18 Task class: StaticHS-StochYields

Brief description

An efficient method for computing quantum yields for a static Hamiltonian that utilizes Monte Carlo sampling for computational speed-up.

Limitations, approximations and comments

- The method requires that there are only two electron spins of spin 1/2.
- This method requires that there is a non-zero number of nuclear spins.
- Only use this method if $M \gg Z$, where M is the number of Monte Carlo samples and Z is the size of the nuclear spin subspace. In contrast, the methodology may exhibit diminished efficiency, with superior alternatives readily available. Generally, this approach should be considered predominantly for large spin systems as the potential errors derived from approximating the integral may be pronounced for smaller system dimensions.
- Ensure the timestep is sufficiently small to prevent significant numerical errors.
- When evaluating quantum yields, please ensure that the *TotalTime* parameter is adequately extended to allow for the complete decay of the 'SpinSystem'.
- The error of approximating the integral scales differently depending on the sampling method. Coherent spin states error scales as $\mathcal{O}(1/\sqrt{M})$. [9] For SUZ sampling method, the error is $\mathcal{O}(1/\sqrt{ZM})$ due to SUZ states having the self-averaging property. [10] SUZ states is default sampling method and coherent states are only implemented for research purposes.
- The overall scaling of the method is $\mathcal{O}(N_t M D \log D)$ where N_t is the number of time-evolution steps and $D = 4Z$. [10]

Parameters

Parameter	Values	Default	Description
AutoSeed	<bool>	false	Defines whether a random choice of a seed number is on. If it is false, then it is checked if a seed number is defined.
Seed	<float>	1	Choosing a seed for the random number generator.
MonteCarloSamp.	<integer>	1000	The number of Monte Carlo states used to approximate the full integral.
InitialState	<i>Singlet</i> , <i>TripletZero</i> , <i>TripletPlus</i> , <i>TripletMinus</i>	<i>Singlet</i>	Choosing the initial state for the electron spins, which can be $ S\rangle$, $ T_0\rangle$, $ T_-\rangle$, or $ T_+\rangle$. The nuclear spins are defined to be in fully mixed state.
SamplingMethod	<i>SUZ</i> , <i>Co-</i> <i>herent</i>	<i>SUZ</i>	The method of sampling state vectors for Monte Carlo approximation. <i>SUZ</i> is the default method due to method's advantageous properties when the full Hilbert space size increases.
TotalTime	<float>	10000	Total integration time (in ns). If no epsilon and total time is defined, then the default is used.
Epsilon	<float>	—	A heuristic that is used to approximate the total time needed for a certain accuracy, which is used to calculate the total time. If totaltime and epsilon are not defined, then the default total time is used.
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
YieldCorrections	<bool>	false	A heuristic that improves the convergence of the method and reduces the total time needed.
PropagationMeth	<i>autoexpm</i> , <i>krylov</i>	<i>autoexpm</i>	The method that is chosen for time-propagation of state vectors. <i>autoexpm</i> needs a further choice of precision and <i>krylov</i> method needs further choice of krylov subspace size and tolerance for restarting.
Precision	<i>double</i> , <i>sin-</i> <i>gle</i> , <i>half</i>	<i>single</i>	Precision of the autoexpm method. The strings relates to the numerical accuracy.
KrylovSize	<integer>	16	The size of the krylov subspace.
KrylovTol	<float>	1×10^{-16}	Tolerance parameter for restarting the krylov subspace calculation. Currently, the method for asymmetric recombination does not use restarting.

Technical details

Solves the Liouville-von Neumann equation for the density operator $\hat{\rho}$:

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}(t)] - \{\hat{K}, \hat{\rho}(t)\} \quad (48)$$

\hat{K} is the Haberkorn reaction operator. Note: the Lindblad form for reactions cannot be used for this method.

The outputs are the yields of state projection operators:

$$\Phi_{\Theta} = k_{\Theta} \int_0^{\infty} P_{\Theta}(t) dt \quad (49)$$

Where, $\Theta = S, T_0, T_-, T_+$, k_Θ is the recombination speed of state Θ and Φ_Θ are the yields of the projection operators \hat{P}_Θ . P_Θ is:

$$P_\Theta(t) = \text{Tr} \left[\hat{P}_\Theta \hat{\rho}(t) \right] \quad (50)$$

We can efficiently compute this trace by stochastically sampling.[11] The method utilizes a set of normalized spin states $|\psi(\xi)\rangle$, parametrized by a set of real values ξ , with which we can resolve the nuclear spin identity operator as:[10]

$$\mathbb{I} = Z \int d\xi p(\xi) |\psi(\xi)\rangle \langle \psi(\xi)| \quad (51)$$

$p(\xi)$ is normalized probability density for ξ . The normalized states are either coherent spin states or SUZ states.

The observable can then be estimated by:

$$P_\Theta(t) = \int d\xi p(\xi) |\Theta, \psi_\xi(t)\rangle \langle \Theta, \psi_\xi(t)| \quad (52)$$

Here $|\Theta, \psi_\xi(0)\rangle = |\Theta\rangle \otimes |\psi(\xi)\rangle$ and the time evolution of the state vector obeys:

$$\frac{d}{dt} |\Theta, \psi_\xi(t)\rangle = \left(-\frac{i}{\hbar} \hat{H} - \hat{K} \right) |\Theta, \psi_\xi(t)\rangle \quad (53)$$

In practise, the integral is approximated by a finite number of Monte Carlo samples. The time-evolution can be carried out by exponential propagation of time-step dt :

$$|\Theta, \psi_\xi(t + dt)\rangle = \exp \left(-\frac{i}{\hbar} \hat{H} dt - \hat{K} dt \right) |\Theta, \psi_\xi(t)\rangle \quad (54)$$

Efficient propagation of these states forms the basis of this method. Due to sparsity of \hat{H} and \hat{K} , this computation scales with $\mathcal{O}(D \log D)$. [9] Making this method ideal for large systems. If the recombination rates for different transitions are symmetric (i.e. equal), then \hat{K} becomes an identity and the recombination rate can be evaluated outside of the exponent and does not have to be considered in the matrix exponential:

$$|\Theta, \psi_\xi(t + dt)\rangle = \exp \left(-\frac{i}{\hbar} \hat{H} dt \right) |\Theta, \psi_\xi(t)\rangle \quad (55)$$

The code automatically identifies if the recombination rates are symmetric.

Moreover, the user can employ an additional heuristic for calculating yields by turning on the *YieldCorrections* flag. The heuristic comes from an observation of how the quantum yields decay when there is symmetric recombination ($k_S = k_T$). Specifically, it can be observed that $1 - \Phi_{RP}(t) \sim e^{-k_S t}$ where t is propagation time and Φ_{RP} radical pair yield. We will denote $1 - \Phi_{RP}$ as ϵ . When calculating yields, we want to evaluate the integral until infinite time, or when all states have fully decayed, i.e. $\epsilon = 0$. At infinite time, then this equation holds $\Phi_S(\infty) + \Phi_T(\infty) = 1$. However, if we only evaluate the integral until time T , then:

$$\Phi_S(T) + \Phi_T(T) = \Phi_{RP}(T) \quad (56)$$

Since, we know that radical pair decays according to exponential we can set $1 - \Phi_{RP}(t) = e^{-k_S t}$:

$$\Phi_S(T) + \Phi_T(T) = 1 - e^{-k_S T} \quad (57)$$

Hence,

$$\frac{\Phi_S(T)}{1 - e^{-k_S T}} + \frac{\Phi_T(T)}{1 - e^{-k_S T}} = 1 \quad (58)$$

Or, in general:

$$\Phi_{\Theta}(T) = \frac{k_{\Theta}}{1 - e^{-k_S T}} \int_0^T P_{\Theta}(t) dt \quad (59)$$

Where we introduce a correction factor:

$$\text{Corr. Factor} = \frac{1}{1 - e^{-k_S T}} \quad (60)$$

The correction factor is derived and optimized for symmetric recombination cases, where it can substantially reduce the required time-propagation duration. It may also be applied in asymmetric cases, though the reduction in propagation time will likely be less significant. For the general case, please note that:

$$\Phi_{\Theta} = \frac{k_{\Theta}}{1 - e^{-k_{max} T}} \int_0^T P_{\Theta}(t) dt \quad (61)$$

Where k_{max} is the fastest recombination process.

Lastly, the total time is estimated by providing a value of ϵ and calculating time with the formula:

$$\epsilon = e^{-k_S t_{max}} \quad (62)$$

$$t_{max} = \frac{\ln(\epsilon^{-1})}{k_S} \quad (63)$$

In practical applications, it has been observed that for symmetric recombination cases, ϵ values as low as 0.5 may yield results with satisfactory graphical accuracy. We advise initiating tests for your system at 0.5, then assessing whether smaller values such as 0.1 or 0.01 offer any significant improvement in accuracy.

When rate is asymmetric, the total time can be approximated with the slowest recombination process, k_{min} :

$$t_{max} = \frac{\ln(\epsilon^{-1})}{k_{min}} \quad (64)$$

Sufficient ϵ values for asymmetric cases depends on the situation and how different individual recombination processes are.

5.19 Task class: StaticHS-StochTimeEvo

Brief description

An efficient method for computing expectation values for a static Hamiltonian that utilizes Monte Carlo sampling for computational speed-up.

Limitations, approximations and comments

- The method requires that there are only two electron spins of spin 1/2.
- This method requires that there is a non-zero number of nuclear spins.
- Only use this method if $M \gg Z$, where M is the number of Monte Carlo samples and Z is the size of the nuclear spin subspace. In contrast, the methodology may exhibit diminished efficiency, with superior alternatives readily available. Generally, this approach should be considered predominantly for large spin systems as the potential errors derived from approximating the integral may be pronounced for smaller system dimensions.
- Ensure the timestep is sufficiently small to prevent significant numerical errors.
- When evaluating quantum yields, please ensure that the *TotalTime* parameter is adequately extended to allow for the complete decay of the 'SpinSystem'.

- The error of approximating the integral scales differently depending on the sampling method. Coherent spin states error scales as $\mathcal{O}(1/\sqrt{M})$. [9] For SUZ sampling method, the error is $\mathcal{O}(1/\sqrt{ZM})$ due to SUZ states having the self-averaging property. [10] SUZ states is default sampling method and coherent states are only implemented for research purposes.
- The overall scaling of the method is $\mathcal{O}(N_t MD \log D)$ where N_t is the number of time-evolution steps and $D = 4Z$. [10]

Parameters

Parameter	Values	Default	Description
AutoSeed	<bool>	false	Defines whether a random choice of a seed number is on. If it is false, then it is checked if a seed number is defined.
Seed	<float>	1	Choosing a seed for the random number generator.
MonteCarloSamp.	<integer>	1000	The number of Monte Carlo states used to approximate the full integral.
InitialState	<i>Singlet, TripletZero, TripletPlus, TripletMinus</i>	<i>Singlet</i>	Choosing the initial state for the electron spins, which can be $ S\rangle$, $ T_0\rangle$, $ T_-\rangle$, or $ T_+\rangle$. The nuclear spins are defined to be in fully mixed state.
SamplingMethod	<i>SUZ, Coherent</i>	<i>SUZ</i>	The method of sampling state vectors for Monte Carlo approximation. SUZ is the default method due to method's advantageous properties when the full Hilbert space size increases.
TotalTime	<float>	10000	Total integration time (in ns). If no epsilon and total time is defined, then the default is used.
Epsilon	<float>	—	A heuristic that is used to approximate the total time needed for a certain accuracy, which is used to calculate the total time. If totaltime and epsilon are not defined, then the default total time is used.
TimeStep	<float>	1.0	Numerical integration timestep (in ns)
PropagationMeth	<i>autoexpm, krylov</i>	<i>autoexpm</i>	The method that is chosen for time-propagation of state vectors. <i>autoexpm</i> needs a further choice of precision and <i>krylov</i> method needs further choice of krylov subspace size and tolerance for restarting.
Precision	<i>double, single, half</i>	<i>single</i>	Precision of the autoexpm method. The strings relates to the numerical accuracy.
KrylovSize	<integer>	16	The size of the krylov subspace.
KrylovTol	<float>	1×10^{-16}	Tolerance parameter for restarting the krylov subspace calculation. Currently, the method for asymmetric recombination does not use restarting.

Technical details

This method is analogous to StaticHS-StochYields; however, the output consists of expectation values instead of yields.

$$P_{\Theta}(t) = \text{Tr} \left[\hat{P}_{\Theta} \hat{\rho}(t) \right] \quad (65)$$

This method does not employ the correction factor to improve the convergence of the quantum yields, as it solely computes the expectation values.

5.20 Task class: StaticHS-DirectYields

Brief description

This method provides an efficient approach for calculating quantum yields in small systems. It employs state vector propagation and directly computes the trace by summing the nuclear spin states.

Limitations, approximations and comments

- The method requires that there are only two electron spins of spin 1/2.
- This method requires that there is a non-zero number of nuclear spins.
- Ensure the timestep is sufficiently small to prevent significant numerical errors.
- When evaluating quantum yields, please ensure that the *TotalTime* parameter is adequately extended to allow for the complete decay of the 'SpinSystem'.
- The overall scaling of the method is $\mathcal{O}(N_t D^2 \log D)$ where N_t is the number of time-evolution steps and $D = 4Z$ and Z is the size of the nuclear spin subspace. [9] The complexity of $D \log D$ stems from the time-propagation of the wavepacket with an exponential matrix. This operation leverages the sparsity of the Hamiltonian, also, requiring only the action of the exponential rather than the entire exponential. An additional complexity of D arises due to the propagation of a wave-packet of size Z to derive expectation values. As a general guideline, restrict the usage of this method to systems of smaller size.

Parameters

Parameter	Values	Default	Description
InitialState	<i>Singlet</i> , <i>TripletZero</i> , <i>TripletPlus</i> , <i>TripletMinus</i>	<i>Singlet</i>	Choosing the initial state for the electron spins, which can be $ S\rangle$, $ T_0\rangle$, $ T_-\rangle$, or $ T_+\rangle$. The nuclear spins are defined to be in fully mixed state.
TotalTime	<float>	10000	Total integration time (in ns). If no epsilon and total time is defined, then the default is used.
Epsilon	<float>	—	A heuristic that is used to approximate the total time needed for a certain accuracy, which is used to calculate the total time. If totaltime and epsilon are not defined, then the default total time is used.
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
YieldCorrections	<bool>	false	A heuristic that improves the convergence of the method and reduces the total time needed.
PropagationMeth	<i>autoexpm</i> , <i>krylov</i>	<i>autoexpm</i>	The method that is chosen for time-propagation of state vectors. <i>autoexpm</i> needs a further choice of precision and <i>krylov</i> method needs further choice of krylov subspace size and tolerance for restarting.
Precision	<i>double</i> , <i>single</i> , <i>half</i>	<i>single</i>	Precision of the autoexpm method. The strings relates to the numerical accuracy.
KrylovSize	<integer>	16	The size of the krylov subspace.
KrylovTol	<float>	1×10^{-16}	Tolerance parameter for restarting the krylov subspace calculation. Currently, the method for asymmetric recombination does not use restarting.

Technical details

Solves the Liouville-von Neumann equation for the density operator $\hat{\rho}$:

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}(t)] - \{\hat{K}, \hat{\rho}(t)\} \quad (66)$$

\hat{K} is the Haberkorn reaction operator. Note: the Lindblad form for reactions cannot be used for this method.

The outputs are the yields of state projection operators:

$$\Phi_{\Theta} = k_{\Theta} \int_0^{\infty} P_{\Theta}(t) dt \quad (67)$$

Where, $\Theta = S, T_0, T_-, T_+$, k_{Θ} is the recombination speed of state Θ and Φ_{Θ} are the yields of the projection operators \hat{P}_{Θ} . P_{Θ} is:

$$P_{\Theta}(t) = \text{Tr} [\hat{P}_{\Theta} \hat{\rho}(t)] \quad (68)$$

The trace is computed directly, summing over possible nuclear spin states: [9]

$$P_{\Theta}(t) = \frac{1}{Z} \sum_{\mathbf{m}_1} \sum_{\mathbf{m}_2} \langle \Theta, \mathbf{M}_1, \mathbf{M}_2; t | \hat{P}_{\Theta} | \Theta, \mathbf{M}_1, \mathbf{M}_2; t \rangle \quad (69)$$

The trace is evaluated in the basis:

$$\{\mathcal{B}\} = \{|\Theta\rangle \otimes |\mathbf{M}_1\rangle \otimes |\mathbf{M}_2\rangle\} \quad (70)$$

$|\mathbf{M}_i\rangle$ is the nuclear spin state of radical i , given by:

$$|\mathbf{M}_i\rangle = |M_{i1}\rangle \otimes |M_{i2}\rangle \otimes \dots \otimes |M_{iN_i}\rangle \quad (71)$$

Where M_{ik} is the projection of the k th nuclear spin in the radical onto the z axis. Here, the state vector is propagated with the exponential matrix

$$|\Theta, \mathbf{M}_1, \mathbf{M}_2; t + dt\rangle = \exp\left(-\frac{i}{\hbar}\hat{H}dt - \hat{K}dt\right) |\Theta, \mathbf{M}_1, \mathbf{M}_2; t\rangle \quad (72)$$

Analogous to the stochastic approach, the propagation process is simplified when dealing with symmetric recombination rates.

The correction factor, as detailed in the `TaskStaticHSStochYields` section, is also employed in this task.

5.21 Task class: `StaticHS-DirectTimeEvo`

Brief description

This method provides an efficient means for expectation value calculations in small systems, leveraging state vector propagation and the direct computation of the trace by summing the nuclear spin states.

Limitations, approximations and comments

- The method requires that there are only two electron spins of spin 1/2.
- This method requires that there is a non-zero number of nuclear spins.
- Ensure the timestep is sufficiently small to prevent significant numerical errors.
- When evaluating quantum yields, please ensure that the `TotalTime` parameter is adequately extended to allow for the complete decay of the 'SpinSystem'.
- The overall scaling of the method is $\mathcal{O}(N_t D^2 \log D)$ where N_t is the number of time-evolution steps and $D = 4Z$ and Z is the size of the nuclear spin subspace. [9] The complexity of $D \log D$ stems from the time-propagation of the wavepacket with an exponential matrix. This operation leverages the sparsity of the Hamiltonian, also, requiring only the action of the exponential rather than the entire exponential. An additional complexity of D arises due to the propagation of a wave-packet of size Z to derive expectation values. As a general guideline, restrict the usage of this method to systems of smaller size.

Parameters

Parameter	Values	Default	Description
InitialState	<i>Singlet</i> , <i>TripletZero</i> , <i>TripletPlus</i> , <i>TripletMinus</i>	<i>Singlet</i>	Choosing the initial state for the electron spins, which can be $ S\rangle$, $ T_0\rangle$, $ T_-\rangle$, or $ T_+\rangle$. The nuclear spins are defined to be in fully mixed state.
TotalTime	<float>	10000	Total integration time (in ns). If no epsilon and total time is defined, then the default is used.
Epsilon	<float>	—	A heuristic that is used to approximate the total time needed for a certain accuracy, which is used to calculate the total time. If totaltime and epsilon are not defined, then the default total time is used.
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
PropagationMeth	<i>autoexpm</i> , <i>krylov</i>	<i>autoexpm</i>	The method that is chosen for time-propagation of state vectors. <i>autoexpm</i> needs a further choice of precision and <i>krylov</i> method needs further choice of krylov subspace size and tolerance for restarting.
Precision	<i>double</i> , <i>single</i> , <i>half</i>	<i>single</i>	Precision of the autoexpm method. The strings relates to the numerical accuracy.
KrylovSize	<integer>	16	The size of the krylov subspace.
KrylovTol	<float>	1×10^{-16}	Tolerance parameter for restarting the krylov subspace calculation. Currently, the method for asymmetric recombination does not use restarting.

Technical details

This method is analogous to StaticHS-DirectYields; however, the output consists of expectation values instead of yields.

$$P_{\Theta}(t) = \text{Tr} \left[\hat{P}_{\Theta} \hat{\rho}(t) \right] \quad (73)$$

This method does not employ the correction factor to improve the convergence of the quantum yields, as it solely computes the expectation values.

5.22 Task class: DynamicHS-StochYields

Brief description

An efficient method for computing quantum yields for a dynamic (time-dependent) Hamiltonian and/or time-dependent recombination rates that utilizes Monte Carlo sampling for computational speed-up.

Limitations, approximations and comments

- The method requires that there are only two electron spins of spin 1/2.
- This method requires that there is a non-zero number of nuclear spins.
- Only use this method if $M \gg Z$, where M is the number of Monte Carlo samples and Z is the size of the nuclear spin subspace. In contrast, the methodology may exhibit diminished efficiency, with superior alternatives readily available. Generally, this approach should be considered predominantly for large spin systems as the potential errors derived from approximating the integral may be pronounced for smaller system dimensions.

- Ensure the timestep is sufficiently small to prevent significant numerical errors.
- When evaluating quantum yields, please ensure that the *TotalTime* parameter is adequately extended to allow for the complete decay of the 'SpinSystem'.
- This method requires that interactions and/or transition rate constants would be time-dependent and would have a trajectory.
- The error of approximating the integral scales differently depending on the sampling method. Coherent spin states error scales as $\mathcal{O}(1/\sqrt{M})$. [9] For SUZ sampling method, the error is $\mathcal{O}(1/\sqrt{ZM})$ due to SUZ states having the self-averaging property. [10] SUZ states is default sampling method and coherent states are only implemented for research purposes.
- The overall scaling of the method is $\mathcal{O}(N_t MD \log D)$ where N_t is the number of time-evolution steps and $D = 4Z$. [10]

Parameters

Parameter	Values	Default	Description
AutoSeed	<bool>	false	Defines whether a random choice of a seed number is on. If it is false, then it is checked if a seed number is defined.
Seed	<float>	1	Choosing a seed for the random number generator.
MonteCarloSamp	<integer>	1000	The number of Monte Carlo states used to approximate the full integral.
InitialState	<i>Singlet, TripletZero, TripletPlus, TripletMinus</i>	<i>Singlet</i>	Choosing the initial state for the electron spins, which can be $ S\rangle$, $ T_0\rangle$, $ T_-\rangle$, or $ T_+\rangle$. The nuclear spins are defined to be in fully mixed state.
SamplingMethod	<i>SUZ, Coherent</i>	<i>SUZ</i>	The method of sampling state vectors for Monte Carlo approximation. SUZ is the default method due to method's advantageous properties when the full Hilbert space size increases.
TotalTime	<float>	10000	Total integration time (in ns).
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
YieldCorrections	<bool>	false	A heuristic that improves the convergence of the method and reduces the total time needed. If transition rate constants are time-dependent, then this property does nothing.
PropagationMeth	<i>autoexpm, krylov</i>	<i>autoexpm</i>	The method that is chosen for time-propagation of state vectors. <i>autoexpm</i> needs a further choice of precision and <i>krylov</i> method needs further choice of krylov subspace size.
Precision	<i>double, single, half</i>	<i>single</i>	Precision of the autoexpm method. The strings relates to the numerical accuracy.
KrylovSize	<integer>	16	The size of the krylov subspace.

Technical details

This method is very similar to the TaskStaticHSStochYields, yet include time-dependent Hamiltonian and/or transition operator. It solves the Liouville-von Neumann equation for the density operator $\hat{\rho}$:

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar} \left[\hat{H}(X(t)), \hat{\rho}(t) \right] - \left\{ \hat{K}(X(t)), \hat{\rho}(t) \right\} \quad (74)$$

Here $X(t)$ is just a set of stochastic parameters and \hat{K} is the Haberkorn reaction operator. :

$$\hat{K}(X(t)) = \frac{k_S(t)}{2} \hat{P}_S + \frac{k_{T_0}(t)}{2} \hat{P}_{T_0} + \frac{k_{T_-}(t)}{2} \hat{P}_{T_-} + \frac{k_{T_+}(t)}{2} \hat{P}_{T_+} = \sum_{\Theta} \frac{k_{\Theta}(t)}{2} \hat{P}_{\Theta} \quad (75)$$

The outputs are the yields of state projection operators:

$$\Phi_{\Theta} = \int_0^{\infty} k_{\Theta}(t) P_{\Theta}(t) dt \quad (76)$$

Just as for the static case, the observables can be approximated by:

$$P_{\Theta}(t) = \int d\xi p(\xi) |\Theta, \psi_{\xi}(t)\rangle \langle \Theta, \psi_{\xi}(t)| \quad (77)$$

Here $|\Theta, \psi_{\xi}(0)\rangle = |\Theta\rangle \otimes |\psi(\xi)\rangle$ and the time evolution of the state vector obeys:

$$\frac{d}{dt} |\Theta, \psi_{\xi}(t)\rangle = \left(-\frac{i}{\hbar} \hat{H}(X(t)) - \hat{K}(X(t)) \right) |\Theta, \psi_{\xi}(t)\rangle \quad (78)$$

The solution to this ODE is:

$$|\Theta, \psi_{\xi}(t+dt)\rangle = \hat{U}(t+dt, t) |\Theta, \psi_{\xi}(t)\rangle \quad (79)$$

Where the propagator \hat{U} is given by:

$$\hat{U}(t_1, t_0) = T \exp \left(\int_{t_0}^{t_1} d\tau \left(-\frac{i}{\hbar} \hat{H}(X(t)) - \hat{K}(X(t)) \right) \right) \quad (80)$$

Here T is time-ordering operator. For small time differences $t_1 - t_0 = dt$, we can use first order Magnus expansion to approximate this time propagation:[12]

$$\begin{aligned} \hat{U}(t+dt, t) &\approx \exp \left(\int_{t_0}^{t_0+dt} d\tau \left(-\frac{i}{\hbar} \hat{H}(X(t)) - \hat{K}(X(t)) \right) \right) \\ &= \exp(-i\Omega(t_0+dt, t_0)dt) \end{aligned} \quad (81)$$

Hence, the state can be evolved by the following generator:

$$\Omega(t_0+dt, t_0) = \frac{1}{\hbar} \hat{H}_0 - \frac{i}{\delta t} \sum_{\Theta} \hat{P}_{\Theta} \int_{t_0}^{t_0+dt} \frac{k_{\Theta}(\tau)}{2} d\tau + \frac{1}{\hbar \delta t} \sum_f \hat{A}_f \int_{t_0}^{t_0+\delta t} f_j(X(\tau)) d\tau \quad (82)$$

Here \hat{A}_f is some spin operator in the Hamiltonian. In principle, the calculation is quite simple and the trajectory of spin operators that is defined should include the calculated $\int_{t_0}^{t_0+\delta t} f_j(X(\tau)) d\tau$ for each time-dependent spin variable at each time-step.

Efficient propagation of these states forms the basis of this method. Due to sparsity of \hat{H} and \hat{K} , this computation scales with $\mathcal{O}(D \log D)$. [9] Making this method ideal for large systems.

Incorporating the correction factor introduced in TaskStaticHSStochYields is also applicable within this methodology, although it necessitates a slightly extended integration time compared to the static case due to persistent time-dependent fluctuations throughout the trajectory. Therefore, it is advisable to employ this correction factor with careful consideration.

5.23 Task class: DynamicHS-StochTimeEvo

Brief Description

An efficient method for computing expectation values for a dynamic (time-dependent) Hamiltonian and/or time-dependent transitions that utilizes Monte Carlo sampling for computational speed-up.

Limitations, approximations and comments

- The method requires that there are only two electron spins of spin 1/2.
- This method requires that there is a non-zero number of nuclear spins.
- This method requires that interactions and/or transition rate constants would be time-dependent and would have a trajectory.
- Only use this method if $M \gg Z$, where M is the number of Monte Carlo samples and Z is the size of the nuclear spin subspace. In contrast, the methodology may exhibit diminished efficiency, with superior alternatives readily available. Generally, this approach should be considered predominantly for large spin systems as the potential errors derived from approximating the integral may be pronounced for smaller system dimensions.
- Ensure the timestep is sufficiently small to prevent significant numerical errors.
- When evaluating quantum yields, please ensure that the *TotalTime* parameter is adequately extended to allow for the complete decay of the 'SpinSystem'.
- The error of approximating the integral scales differently depending on the sampling method. Coherent spin states error scales as $\mathcal{O}(1/\sqrt{M})$. [9] For SUZ sampling method, the error is $\mathcal{O}(1/\sqrt{ZM})$ due to SUZ states having the self-averaging property. [10] SUZ states is default sampling method and coherent states are only implemented for research purposes.
- The overall scaling of the method is $\mathcal{O}(N_t M D \log D)$ where N_t is the number of time-evolution steps and $D = 4Z$. [10]

Parameters

Parameter	Values	Default	Description
AutoSeed	<bool>	false	Defines whether a random choice of a seed number is on. If it is false, then it is checked if a seed number is defined.
Seed	<float>	1	Choosing a seed for the random number generator.
MonteCarloSamp	<integer>	1000	The number of Monte Carlo states used to approximate the full integral.
InitialState	<i>Singlet,</i> <i>TripletZero,</i> <i>TripletPlus,</i> <i>TripletMi-</i> <i>minus</i>	<i>Singlet</i>	Choosing the initial state for the electron spins, which can be $ S\rangle$, $ T_0\rangle$, $ T_-\rangle$, or $ T_+\rangle$. The nuclear spins are defined to be in fully mixed state.
SamplingMethod	<i>SUZ,</i> <i>Co-</i> <i>herent</i>	<i>SUZ</i>	The method of sampling state vectors for Monte Carlo approximation. <i>SUZ</i> is the default method due to method's advantageous properties when the full Hilbert space size increases.
TotalTime	<float>	10000	Total integration time (in ns).
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
PropagationMeth	<i>autoexpm,</i> <i>krylov</i>	<i>autoexpm</i>	The method that is chosen for time-propagation of state vectors. <i>autoexpm</i> needs a further choice of precision and <i>krylov</i> method needs further choice of krylov subspace size.
Precision	<i>double,</i> <i>sin-</i> <i>gle.</i> <i>half</i>	<i>single</i>	Precision of the autoexpm method. The strings relates to the numerical accuracy.
KrylovSize	<integer>	16	The size of the krylov subspace.

Technical details

This method is analogous to DynamicHS-StochYields; however, the output consists of expectation values instead of yields.

$$P_{\Theta}(t) = \text{Tr} \left[\hat{P}_{\Theta} \hat{\rho}(t) \right] \quad (83)$$

This method does not employ the correction factor to improve the convergence of the quantum yields, as it solely computes the expectation values.

5.24 Task class: DynamicHS-DirectYields

Brief description

An efficient method for small systems to calculate quantum yields utilizing state vector propagation and computing the trace directly by the sum of nuclear spin states.

Limitations, approximations and comments

- The method requires that there are only two electron spins of spin 1/2.
- This method requires that there is a non-zero number of nuclear spins.
- Ensure the timestep is sufficiently small to prevent significant numerical errors.

- When evaluating quantum yields, please ensure that the *TotalTime* parameter is adequately extended to allow for the complete decay of the 'SpinSystem'.
- This method requires that interactions and/or transition rate constants would be time-dependent and would have a trajectory.
- The overall scaling of the method is $\mathcal{O}(N_t D^2 \log D)$ where N_t is the number of time-evolution steps and $D = 4Z$ and Z is the size of the nuclear spin subspace. [9] The complexity of $D \log D$ stems from the time-propagation of the wavepacket with an exponential matrix. This operation leverages the sparsity of the Hamiltonian, also, requiring only the action of the exponential rather than the entire exponential. An additional complexity of D arises due to the propagation of a wave-packet of size Z to derive expectation values. As a general guideline, restrict the usage of this method to systems of smaller size.

Parameters

Parameter	Values	Default	Description
InitialState	<i>Singlet</i> , <i>TripletZero</i> , <i>TripletPlus</i> , <i>TripletMinus</i>	<i>Singlet</i>	Choosing the initial state for the electron spins, which can be $ S\rangle$, $ T_0\rangle$, $ T_-\rangle$, or $ T_+\rangle$. The nuclear spins are defined to be in fully mixed state.
TotalTime	<float>	10000	Total integration time (in ns).
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
YieldCorrections	<bool>	false	A heuristic that improves the convergence of the method and reduces the total time needed. If transition rate constants are time-dependent, then this property does nothing.
PropagationMeth	<i>autoexpm</i> , <i>krylov</i>	<i>autoexpm</i>	The method that is chosen for time-propagation of state vectors. <i>autoexpm</i> needs a further choice of precision and <i>krylov</i> method needs further choice of krylov subspace size.
Precision	<i>double</i> , <i>single</i> , <i>half</i>	<i>single</i>	Precision of the autoexpm method. The strings relates to the numerical accuracy.
KrylovSize	<integer>	16	The size of the krylov subspace.

Technical details

This method is very similar to TaskStaticHSDirectYields, yet includes time-dependent terms in the dynamics. It solves the Liouville-von Neumann equation for the density operator $\hat{\rho}$:

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar} \left[\hat{H}(X(t), \hat{\rho}(t)) \right] - \left\{ \hat{K}(X(t), \hat{\rho}(t)) \right\} \quad (84)$$

Here $X(t)$ is just a set of stochastic parameters and \hat{K} is the Haberkorn reaction operator. :

$$\hat{K}(X(t)) = \frac{k_S(t)}{2} \hat{P}_S + \frac{k_{T_0}(t)}{2} \hat{P}_{T_0} + \frac{k_{T_-}(t)}{2} \hat{P}_{T_-} + \frac{k_{T_+}(t)}{2} \hat{P}_{T_+} = \sum_{\Theta} \frac{k_{\Theta}(t)}{2} \hat{P}_{\Theta} \quad (85)$$

The outputs are the yields of state projection operators:

$$\Phi_{\Theta} = \int_0^{\infty} k_{\Theta}(t) P_{\Theta}(t) dt \quad (86)$$

As for the static case, the trace can be computed as:

$$P_{\Theta}(t) = \frac{1}{Z} \sum_{\mathbf{m}_1} \sum_{\mathbf{m}_2} \langle \Theta, \mathbf{M}_1, \mathbf{M}_2; t | \hat{P}_{\Theta} | \Theta, \mathbf{M}_1, \mathbf{M}_2; t \rangle \quad (87)$$

The trace is evaluated in the basis:

$$\{\mathcal{B}\} = \{|\Theta\rangle \otimes |\mathbf{M}_1\rangle \otimes |\mathbf{M}_2\rangle\} \quad (88)$$

$|\mathbf{M}_i\rangle$ is the nuclear spin state of radical i , given by:

$$|\mathbf{M}_i\rangle = |M_{i1}\rangle \otimes |M_{i2}\rangle \otimes \dots \otimes |M_{iN_i}\rangle \quad (89)$$

Where M_{ik} is the projection of the k th nuclear spin in the radical onto the z axis. Here, the state vector is propagated as:

$$\frac{d}{dt} |\Theta, \mathbf{M}_1, \mathbf{M}_2; t\rangle = \left(-\frac{i}{\hbar} \hat{H}(X(t)) - \hat{K}(X(t)) \right) |\Theta, \mathbf{M}_1, \mathbf{M}_2; t\rangle \quad (90)$$

The solution to this ODE is:

$$|\Theta, \mathbf{M}_1, \mathbf{M}_2; t + dt\rangle = \hat{U}(t + dt, t) |\Theta, \mathbf{M}_1, \mathbf{M}_2; t\rangle \quad (91)$$

Where the propagator \hat{U} is given by:

$$\hat{U}(t_1, t_0) = T \exp \left(\int_{t_0}^{t_1} d\tau \left(-\frac{i}{\hbar} \hat{H}(X(t)) - \hat{K}(X(t)) \right) \right) \quad (92)$$

Here T is time-ordering operator. For small time differences $t_1 - t_0 = dt$, we can use first order Magnus expansion to approximate this time propagation:[12]

$$\begin{aligned} \hat{U}(t + dt, t) &\approx \exp \left(\int_{t_0}^{t_0+dt} d\tau \left(-\frac{i}{\hbar} \hat{H}(X(t)) - \hat{K}(X(t)) \right) \right) \\ &= \exp(-i\Omega(t_0 + dt, t_0)dt) \end{aligned} \quad (93)$$

Hence, the state can be evolved by the following generator:

$$\Omega(t_0 + dt, t_0) = \frac{1}{\hbar} \hat{H}_0 - \frac{i}{\delta t} \sum_{\Theta} \hat{P}_{\Theta} \int_{t_0}^{t_0+dt} \frac{k_{\Theta}(\tau)}{2} d\tau + \frac{1}{\hbar \delta t} \sum_f \hat{A}_f \int_{t_0}^{t_0+\delta t} f_j(X(\tau)) d\tau \quad (94)$$

Here \hat{A}_f is some spin operator in the Hamiltonian. In principle, the calculation is quite simple and the trajectory of spin operators that is defined should include the calculated $\int_{t_0}^{t_0+\delta t} f_j(X(\tau)) d\tau$ for each time-dependent spin variable at each time-step.

Incorporating the correction factor introduced in TaskStaticHSStochYields is also applicable within this methodology, although it necessitates a slightly extended integration time compared to the static case due to persistent time-dependent fluctuations throughout the trajectory. Therefore, it is advisable to employ this correction factor with careful consideration.

5.25 Task class: DynamicHS-DirectTimeEvo

Brief description

An efficient method for small systems to calculate expectation values utilizing state vector propagation and computing the trace directly by the sum of nuclear spin states.

Limitations, approximations and comments

- The method requires that there are only two electron spins of spin 1/2.
- This method requires that there is a non-zero number of nuclear spins.
- Ensure the timestep is sufficiently small to prevent significant numerical errors.
- When evaluating quantum yields, please ensure that the *TotalTime* parameter is adequately extended to allow for the complete decay of the 'SpinSystem'.
- This method requires that interactions and/or transition rate constants would be time-dependent and would have a trajectory.
- The overall scaling of the method is $\mathcal{O}(N_t D^2 \log D)$ where N_t is the number of time-evolution steps and $D = 4Z$ and Z is the size of the nuclear spin subspace. [9] The complexity of $D \log D$ stems from the time-propagation of the wavepacket with an exponential matrix. This operation leverages the sparsity of the Hamiltonian, also, requiring only the action of the exponential rather than the entire exponential. An additional complexity of D arises due to the propagation of a wave-packet of size Z to derive expectation values. As a general guideline, restrict the usage of this method to systems of smaller size.

Parameters

Parameter	Values	Default	Description
InitialState	<i>Singlet</i> , <i>TripletZero</i> , <i>TripletPlus</i> , <i>TripletMinus</i>	<i>Singlet</i>	Choosing the initial state for the electron spins, which can be $ S\rangle$, $ T_0\rangle$, $ T_-\rangle$, or $ T_+\rangle$. The nuclear spins are defined to be in fully mixed state.
TotalTime	<float>	10000	Total integration time (in ns).
TimeStep	<float>	1.0	Numerical integration timestep (in ns).
PropagationMeth	<i>autoexpm</i> , <i>krylov</i>	<i>autoexpm</i>	The method that is chosen for time-propagation of state vectors. <i>autoexpm</i> needs a further choice of precision and <i>krylov</i> method needs further choice of krylov subspace size.
Precision	<i>double</i> , <i>single</i> , <i>half</i>	<i>single</i>	Precision of the autoexpm method. The strings relates to the numerical accuracy.
KrylovSize	<integer>	16	The size of the krylov subspace.

Technical details

This method is analogous to DynamicHS-DirectYields; however, the output consists of expectation values instead of yields.

$$P_{\Theta}(t) = \text{Tr} \left[\hat{P}_{\Theta} \hat{\rho}(t) \right] \quad (95)$$

This method does not employ the correction factor to improve the convergence of the quantum yields, as it solely computes the expectation values.

5.26 Task class: StaticSS-PumpProbe

Brief description

Superoperator space (Liouville space) method that calculates the quantum yield over a given timespan (instead of over all time). The differences to the StaticSS-TimeEvolution task is that the quantum yield integral is exactly and directly evaluated between two time points; the *PumpTime* and the *ProbeTime*.

Limitations, approximations and comments

- Superoperator space methods scale much worse with system size than native Hilbert space methods (in terms of both memory and speed).
- Ignores time-dependencies.

Parameters

Parameter	Values	Default	Description
TransitionYields	<bool>	false	Whether to calculate quantum yields per transition (true) or per State defined in the SpinSystem (false).
ReactionOperator	<i>Haberkorn</i> , <i>Lindblad</i>	<i>Haberkorn</i>	The default reaction operator type to use for this task. This is ignored for Transition objects where the reaction operator type is specified.
PumpTime	<i>float</i>	0	Time of the pump pulse (in ns).
ProbeTime	<i>float</i>	10000	Time of the probe pulse (in ns).

Technical details

Solves the Liouville-von Neumann equation for the density operator ρ , in superoperator space (Liouville space):

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\hat{\mathbf{H}}\rho + \hat{\mathbf{K}}\rho = \hat{\mathbf{A}}\rho, \quad (96)$$

where $\hat{\mathbf{H}}$ is the Hamiltonian superoperator, $\hat{\mathbf{K}}$ is the superoperator representing reactions (either Haberkorn or Lindblad form), and $\hat{\mathbf{A}}$ is the total Liouvillian superoperator.

The quantum yield is solved by:

$$\Phi_i = \int_{t_1}^{t_2} Tr[\mathbf{P}_i\rho(t),]dt, \quad (97)$$

where \mathbf{P}_i is the respective state projection operator defined in a *StateObject*. Given the system is being evaluated at discrete times, we must allow the density operator to evolve:

$$\hat{\rho}(t) = exp(-\hat{\mathbf{A}}t)\hat{\rho}(0). \quad (98)$$

There are two possibilities for calculation of the quantum yields; by default the yields are just calculated for each State object defined in the SpinSystem:

$$\Phi_i = \int_{t_1}^{t_2} Tr[\mathbf{P}_i exp(-\hat{\mathbf{A}}t)\rho(t)]dt = Tr[\mathbf{P}_i\hat{\mathbf{A}}^{-1}(exp(-\hat{\mathbf{A}}t_1) - exp(-\hat{\mathbf{A}}t_2))\rho(0)]. \quad (99)$$

Note that there is no rate constant involved here; thus this is not a “proper yield”. The other possibility is when the *TransitionYields* parameter is set to *true*:

$$\Phi_j = k_j \int_{t_1}^{t_2} Tr[\mathbf{P}_j exp(-\hat{\mathbf{A}}t)\rho(t)]dt = k_j Tr[\mathbf{P}_j\hat{\mathbf{A}}^{-1}(exp(-\hat{\mathbf{A}}t_1) - exp(-\hat{\mathbf{A}}t_2))\rho(0)]. \quad (100)$$

Here the index j refers to the *SourceState* parameter of a *TransitionObject* (which is also a *StateObject*, and hence \mathbf{P}_j is the projection onto that state) and k_j the rate of the transition. The relationship as follows is used to evaluate the integral:

$$\int \exp(-\hat{\mathbf{A}}t) dt = -\hat{\mathbf{A}}^{-1} \exp(-\hat{\mathbf{A}}t). \quad (101)$$

Brief description

Hilbert space method that calculates the quantum yield over a given timespan (instead of over all time). The differences to the StaticHS-TimeEvolution task is that the quantum yield integral is exactly and directly evaluated between two time points; the *PumpTime* and the *ProbeTime*.

Limitations, approximations and comments

- The interactions should be time-independent as they are not evaluated throughout the calculation.
- Only a single spin-independent reaction is present (i.e. “symmetric decay” as all states decay with same rate).
- This is an exact calculation of the quantum yield over a given time period.

Parameters

Parameter	Values	Default	Description
DecayRate ReactionRate RateConstant	<float>	0.001	The spin-independent rate constant to use (in ns ⁻¹). If the rate constant is not specified, the rate of the first defined Transition object in each spin system is used. If no Transition objects are defined in a spin system, the default value will be used.
PumpTime	<i>float</i>	0	Time of the pump pulse (in ns).
ProbeTime	<i>float</i>	10000	Time of the probe pulse (in ns).

Technical details

Calculates the quantum yield by solving the equation:

$$\Phi_i = k \int_{t_1}^{t_2} \mathbf{P}_i(t) \exp(-kt) dt \quad (102)$$

where,

$$\mathbf{P}_i(t) = \frac{1}{M} \text{Tr}[\exp(-i\hat{\mathbf{H}}t) \hat{P}_s \exp(i\hat{\mathbf{H}}t) \hat{P}_i] \quad (103)$$

\hat{P}_i has been used to represent the initial state distribution.

To solve the given equation we must first rearrange the equation to a soluble form:

$$\mathbf{P}_i(t) = \frac{1}{M} \sum_j \sum_k \sum_m \sum_n (\exp(-i\hat{\mathbf{H}}t))_{jk} (\hat{P}_s)_{km} (\exp(i\hat{\mathbf{H}}t))_{mn} (\hat{P}_i)_{nj}, \quad (104)$$

$$\mathbf{P}_i(t) = \frac{1}{M} \sum_j \sum_m \exp(-ih_j t) (\hat{P}_s)_{jm} \exp(ih_m t) (\hat{P}_i)_{mj}, \quad (105)$$

$$\mathbf{P}_i(t) = \frac{1}{M} \sum_j \sum_m \exp(i(h_m - h_j)t) (\hat{P}_s)_{jm} (\hat{P}_i)_{mj}, \quad (106)$$

$$\omega_{mj} = h_m - h_j, \quad (107)$$

$$\mathbf{P}_i(t) = \frac{1}{M} \sum_j \sum_m \exp(-i\omega_{mj}t) (\hat{P}_s)_{jm} (\hat{P}_i)_{mj}, \quad (108)$$

this is now in a form that can be easily solved:

$$\Phi_i = k \int_{t_1}^{t_2} \mathbf{P}_i(t) \exp(-kt) dt = \frac{k}{M} \sum_j \sum_m (\hat{P}_s)_{jm} (\hat{P}_i)_{mj} \int_{t_1}^{t_2} \cos(\omega_{mj}t) \exp(-kt) dt, \quad (109)$$

the integral is exactly, and simply, soluble:

$$\int_{t_1}^{t_2} \cos(\omega_{mj}t) \exp(-kt) dt = \exp(-kt_2) f(\omega_{mj}, t_2) - \exp(-kt_1) f(\omega_{mj}, t_1), \quad (110)$$

where,

$$f(\omega_{mj}, t) = \frac{\omega_{mj} \sin(\omega_{mj}t) - k \cos(\omega_{mj}t)}{\omega_{mj}^2 + k^2}. \quad (111)$$

Finally we arrive to the following expression:

$$\Phi_i = \frac{k}{M} \sum_j \sum_m (\hat{P}_s)_{jm} (\hat{P}_i)_{mj} [\exp(-kt_2) f(\omega_{mj}, t_2) - \exp(-kt_1) f(\omega_{mj}, t_1)]. \quad (112)$$

6 Preprocessor directives

Similar to the C programming language, the MolSpin input format allows the use of a variety of preprocessor directives. Such directives are prefixed with the symbol `#`, but the directives are slightly more restricted than in C since they cannot be invoked inside the definition of objects (e.g. Spin or Transition objects), and the *include* directive cannot be used within an object group either (spinsystem, settings or run section). The main reason for this limitation is that the directives are read “on the fly” while parsing the input files, rather than an actual preprocessing (although the “preprocessor directive” term was adopted for its similarity with C).

6.1 `#include <filename>`

The *include* directive can be used to split your input into multiple files. You can, for example, define your spin system in one file, and use it for different purposes by defining settings and run sections in other files including your spin system file.

Note that MolSpin keeps track of the files that are included, such that repeated or recursive inclusion of the same files is prevented.

6.2 `#define <name> <value (optional)>`

If used without a value, the define directive can only be used by other directives such as *ifdef* or *ifndef*. This way it is possible to e.g. create so-called *guards*⁶ or to easily switch several lines on/off by commenting in/out a define directive.

If a value was also supplied, any occurrence of the defined name in any object will be replaced by the value. *Note the limitation here:* Only the *contents of objects*⁷ can be replaced by the defined value. If you need to use spaces in either the name or value of your definition you can write them as strings, i.e. between quotes (“this is a string”), but remember that strings are always treated as case sensitive in MolSpin!

6.3 `#undef <name>`

Removes a definition (regardless of whether the definition had an associated value). The definition is valid from the point in the input file where the define directive is made and until the *undef* directive.

6.4 `#ifdef <name>`

Short for “if defined”. This directive does nothing if the specified name was defined previously by a define directive, but if the specified name was *not* defined previously (or if the definition was removed previously by *undef*), all the subsequent contents of the input file is ignored until an *endif* or *else* directive is encountered.

Should always have a corresponding *endif* directive.

6.5 `#ifndef <name>`

Short for “if not defined”. Same as the *ifdef* directive except that the following contents are instead ignored if the specified name *was* defined previously. Thus this is the logical negation of the *ifdef* directive (i.e. opposite behaviour).

Should always have a corresponding *endif* directive.

⁶*Guards* is a standard technique in C or C++ programming to prevent inclusion of the same file twice by putting all the code between *ifndef* and *endif*, where the first line after *ifndef* makes the define directive that was just checked. This is used in all the header (.h) files of the MolSpin source code.

⁷Objects refer to Spins, Interactions, Transitions, etc. but not object groups (SpinSystems, Settings and Run Section), and the define directives are only valid for their *contents*, i.e. everything between { and }. Thus a define directive has no effect on e.g. the name of a Spin object.

6.6 #else

This directive can be used in between an *ifdef* or *ifndef* directive and its corresponding *endif* directive. Either the input between *ifdef/ifndef* and *else* or between *else* and *endif* will be ignored, i.e. the *else* directive “switches” between ignoring and not ignoring the lines between *ifdef/ifndef* and its corresponding *endif*.

6.7 #endif

Denotes the end of an *ifdef* or *ifndef* directive.

References

- [1] P. Gaspard and M. Nagaoka. Slippage of initial conditions for the redfield master equation. *The Journal of Chemical Physics*, 111(13):5668–5675, 1999.
- [2] Maurice Goldman. Formal theory of spin–lattice relaxation. *Journal of Magnetic Resonance*, 149(2):160–187, 2001.
- [3] Heinz-Peter Breuer, Francesco Petruccione, et al. *The theory of open quantum systems*. Oxford University Press on Demand, 2002.
- [4] Michael Mehring and Volker Achim Weberruss. *Object-oriented magnetic resonance: classes and objects, calculations and computations*. Elsevier, 2012.
- [5] Susannah Worster, Daniel R Kattnig, and PJ Hore. Spin relaxation of radicals in cryptochrome and its role in avian magnetoreception. *The Journal of Chemical Physics*, 145(3):035104, 2016.
- [6] Daniel R. "Kattnig, Ilia A. Solov'yov, and P J" Hore. Electron spin relaxation in cryptochrome-based magnetoreception. *Phys. Chem. Chem. Phys.*, 18:12443–12456, 2016.
- [7] Conrad Sanderson and Ryan Curtin. Armadillo: a template-based c++ library for linear algebra. *Journal of Open Source Software*, 1(2):26, 2016.
- [8] Rohit Chandra, Leo Dagum, David Kohr, Ramesh Menon, Dror Maydan, and Jeff McDonald. *Parallel programming in OpenMP*. Morgan kaufmann, 2001.
- [9] Alan M. Lewis, Thomas P. Fay, and David E. Manolopoulos. An efficient quantum mechanical method for radical pair recombination reactions. *The Journal of Chemical Physics*, 145(24):244101, 2016.
- [10] T. P. Fay, L. P. Lindoy, and D. E. Manolopoulos. Spin relaxation in radical pairs from the stochastic Schrödinger equation. *The Journal of Chemical Physics*, 154(8):084121, feb 28 2021.
- [11] Alexander Weiße, Gerhard Wellein, Andreas Alvermann, and Holger Fehske. The kernel polynomial method. *Rev. Mod. Phys.*, 78:275–306, Mar 2006.
- [12] W. Magnus. On the exponential solution of differential equations for a linear operator. *Communications on Pure and Applied Mathematics*, 7(4):649–673, 11 1954.